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Numerical study on the effects of uneven bottom topography on freak waves

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ABSTRACT

A numerical model is built by using an improved VOF method coupled with an incompressible Navier–Stokes solver. Exploiting the model, the freak wave formation due to the dispersive focusing mechanism is investigated numerically without uneven bottoms and in presence of uneven bottoms. During the freak wave transformation over an uneven bottom in finite water, combined effects of shoaling, refraction and reflection can modify the external characteristics of freak waves, and also can complicate the energy transfers. Furthermore, wavelet analysis method is adopted to analyze the behavior of the instantaneous energy structure of freak waves. It is found that when the bottoms vary in height, the external characteristic parameters and high frequency energy show a similar trend, but the value may be quite different due to the difference in local characteristic of the bottom.

1. Introduction

Freak wave is a type of extremely large transient water wave, being close to breaking and of asymmetry in both vertical and horizontal direction. In the time-frequency spectrum of freak waves, strong energy density is instantly surged and seemingly carried over to the high frequency components at the instant the freak wave occurs (Liu and Mori, 2000), therefore severe damages to vessels, maritime structures and other facilities in the ocean are usually caused. Such waves have been observed in a large number of basins around the world, in deep or shallow waters, with or without currents (Kharif and Pelinovsky, 2003; Lopatoukhin and Boukhanovsky, 2004; Monbaliu and Toffoli, 2003), and the popular Rayleigh distribution cannot precisely predict the probability of occurrence of freak waves due to the non-linear effects (Stansell, 2004, 2005; Chien et al., 2002).

The previous studies have shown that several mechanisms have been suggested to explain the formation of freak waves in various environments. Among them one can mention the dispersive focusing, wave–current interaction, atmospheric forcing, spatial (geometrical) focusing, non-linear self-focusing of wave energy, and non-linear wave–wave interaction (Chien et al., 2002; Kharif and Pelinovsky, 2003; Lopatoukhin and Boukhanovsky, 2004).

Wu and Yao (2004) reported the results of laboratory measurements on limiting freak waves in the presence of currents. It is found that strong opposing currents inducing partial wave blocking significantly elevate the limiting steepness and asymmetry of freak waves.

Touboul et al. (2006) experimentally and numerically investigated the direct effect of the wind on a freak wave event generated by means of a dispersive focusing mechanism. The results suggest that the duration of the freak wave event increases with the wind velocity, and the point where the waves merge has a shift in the downstream direction, which is due to the action of the current induced by the wind.

Peterson et al. (2003) presented that non-linear interactions of solitonic waves in the framework of the Kadomtsev–Petviashvili equation may result in particularly high and steep waves resembling the freak waves, and it may be a generic source of freak waves in areas of moderate depth.

Overall, these studies have provided us a good understanding to the influences on the external characteristics of freak waves. However, insight into the internal structure of freak waves plays an important role in interpreting the physical mechanism. Walker et al. (2004) investigated the non-linear characteristics of freak waves based on Fourier power spectrum. It is found that the field data exhibits an anomalous set-up for the New Year wave, whereas all the other large waves show a local set-down. The conventional Fourier method has provided substantial insight into freak wave phenomena, but the Fourier power spectrum, as a time-averaged description of wave energy, is inappropriate for characterizing non-stationary signals. Wavelet method has been proven to be a powerful tool for analyzing localized variations of power within a time series. By decomposing a time series into time-frequency space, one is able to determine both the dominant modes of variability and how those modes vary in time. By using this method, the analysis on time-frequency energy spectrum of simulated and field observed freak waves has been presented by the authors (Cherneva and Soares, 2008; Chien et al., 2002; Cui and Zhang, 2011; Mori et al., 2002). It is found that a well-defined

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freak wave can be readily identified from the wavelet spectrum where strong energy density in the spectrum is instantly surged and seemingly carried over to the high frequency components at the instant the freak wave occurs. Wavelet analysis method has better performance than the popular Fourier technique.

One believes that the bottom topography plays important role in modifying wave form and propagation. During the wave transformation from the deep-water to shallow-water over uneven bottoms, combined effects of shoaling, refraction, diffraction, and reflection can result in bending, overturning and breaking waves (Biausser et al., 2003; Choi and Wu, 2006; Grilli et al., 2001). In addition, spatial (geometrical) focusing is one of the possible physical mechanisms of freak waves. To our knowledge, the internal features and evolution behavior of freak waves in presence of uneven bottoms need further examination.

In this paper, a numerical model is built by using an improved VOF method (Ren and Wang, 2004) and the governing equations are the Reynolds-averaged N–S (RANS) equations, closed by the two-equation k–ε turbulence model. The component waves focusing method is adopted for freak wave formation, which is achieved through using a wave-maker to generate waves at one extremity of the numerical tank and the motion of the wave-maker is prescribed according to an improved superposition model (Kriebel, 2000). The uneven bottoms can be characterized by including a partial bottom-cell treatment. By using the current model, freak waves without uneven bottoms and in presence of slope and curved topography have been simulated to analyze the effects of the uneven bottoms on the external features and internal energy structure of freak waves.

2. Mathematical model

2.1. Governing equations

When freak waves propagate over uneven bottoms, they may break with turbulent fluctuation of the water particles due to the wave–bottom interaction, which must be accounted for by proper turbulence model. Therefore, the two-dimensional continuity equation and the Reynolds-averaged N–S equations are used as the governing equations, closed by the two-equation k–ε turbulence model.

Continuity equation:

$$\frac{\partial}{\partial x}(\theta u)+\frac{\partial}{\partial y}(\theta v)=0 \qquad (1)$$

Reynolds time-averaged equations:

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-\frac{1}{\rho}\frac{\partial p}{\partial x}+g_x+\nu\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right)+\nu_t\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right)+2\frac{\partial \nu_t}{\partial x}\frac{\partial u}{\partial x}+\frac{\partial \nu_t}{\partial y}\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right)-\frac{2}{3}\frac{\partial k}{\partial x} \qquad (2)$$

$$\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}=-\frac{1}{\rho}\frac{\partial p}{\partial y}+g_y+\nu\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right)+\nu_t\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right)+2\frac{\partial \nu_t}{\partial y}\frac{\partial v}{\partial y}+\frac{\partial \nu_t}{\partial x}\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right)-\frac{2}{3}\frac{\partial k}{\partial y} \qquad (3)$$

Two-equation $k-\varepsilon$ turbulence model:

$$\frac{\partial k}{\partial t}+u\frac{\partial k}{\partial x}+v\frac{\partial k}{\partial y}=\left(\nu+\frac{\nu_t}{\sigma_k}\right)\left(\frac{\partial^2 k}{\partial x^2}+\frac{\partial^2 k}{\partial y^2}\right)+\frac{1}{\sigma_k}\left(\frac{\partial \nu_t}{\partial x}\frac{\partial k}{\partial x}+\frac{\partial \nu_t}{\partial y}\frac{\partial k}{\partial y}\right)+2\nu_t\left[\left(\frac{\partial u}{\partial x}\right)^2+\left(\frac{\partial v}{\partial y}\right)^2\right]+\nu_t\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right)^2-\varepsilon \qquad (4)$$

$$\frac{\partial \varepsilon}{\partial t}+u\frac{\partial \varepsilon}{\partial x}+v\frac{\partial \varepsilon}{\partial y}=\left(\nu+\frac{\nu_t}{\sigma_\varepsilon}\right)\left(\frac{\partial^2 \varepsilon}{\partial x^2}+\frac{\partial^2 \varepsilon}{\partial y^2}\right)+\frac{1}{\sigma_\varepsilon}\left(\frac{\partial \nu_t}{\partial x}\frac{\partial \varepsilon}{\partial x}+\frac{\partial \nu_t}{\partial y}\frac{\partial \varepsilon}{\partial y}\right)+2C_{\varepsilon 1}\frac{\varepsilon}{k}+\nu_t\left[\left(\frac{\partial u}{\partial x}\right)^2+\left(\frac{\partial v}{\partial y}\right)^2\right]+C_{\varepsilon 1}\frac{\varepsilon}{k}\nu_t\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right)^2-C_{\varepsilon 2}\frac{\varepsilon^2}{k} \qquad (5)$$

where, u and v are velocity components in x- and y-directions, respectively; θ is the partial-cell parameter, which is independent of time and has a value between 0 and 1 depending on whether the point is inside an obstacle or in the fluid; p is the pressure; ρ is the fluid density; g is the gravitational acceleration; ν is the coefficient of kinematic viscosity; $\nu_t=C_u\,(k^2/\varepsilon)$ is the coefficient of turbulent viscosity; k is the turbulent kinetic energy; ε is the turbulent kinetic energy dissipation rate; C_u, σ_k, σ_ε, $C_{\varepsilon 1}$ and $C_{\varepsilon 2}$ are the empirical constants recommended in the literature (Rodi, 1993). In this work, the following standard values are used: $C_u=0.09$, $\sigma_k=1.0$, $\sigma_\varepsilon=1.3$, $C_{\varepsilon 1}=1.43$, $C_{\varepsilon 2}=1.92$.

2.2. Numerical method

The VOF method is known for its capacity to simulate free surface flow. This is made possible by means of a fluid fraction $F(x, y, t)$, which has a value between zero and unity, representing the volume fraction of a cell occupied by fluid. Thus, a cell full of fluid is reflected by $F=1$, while an empty cell will have $F=0$. A cell that is either intersected by a free surface or contains voids will be partially filled with fluid and has a value of F between zero and unity. Furthermore, a free surface cell can be identified being a cell with a non-zero F and having at least one neighboring cell where $F=0$. The time variation of this function is governed by

$$\frac{\partial(\theta F)}{\partial t}+\frac{\partial(\theta u F)}{\partial x}+\frac{\partial(\theta v F)}{\partial y}=0 \qquad (6)$$

The variables are solved for from a finite-difference approximation of the governing equations. On the discretization of the advection items, a third-order upwind scheme is used when dealing with inner points, and a hybrid scheme combining first-order upwind and second-order central differences is used when dealing with boundary points. The second-order central difference scheme is used for the viscous terms (Ren and Wang, 2004). The solution algorithm as detailed in the original VOF method is employed (Hirt and Nichols, 1981).

This method allows for simulation of breaking and post-breaking waves (Biausser et al., 2003) as well as steep waves that have high velocity near the surface (freak waves).

2.3. Boundary conditions

2.3.1. Boundary conditions for the freak wave-maker

The improved superposition model is used to generate freak waves. In the model, an extreme transient wave is embedded into a random wave train, based on a partitioning of the total wave energy with one part of the energy going into the underlying random sea and the other into the focused transient wave (Kriebel, 2000).

The model can be expressed as

$$\eta(x,t)=\eta_1(x,t)+\eta_2(x,t)=\sum_{i=1}^{M}a_{1i}\cos(k_i x-\omega_i^* t+\varepsilon_i)+\sum_{i=1}^{M}a_{2i}\cos[k_i(x-x_c)-\omega_i^*(t-t_c)] \qquad (7)$$

where η is the surface elevation at a distance x from the wave generator in the wave tank; M is the number of component wave;