Ocean Engineering 50 (2012) 72–82

Contents lists available at SciVerse ScienceDirect

Ocean Engineering

journal homepage: www.elsevier.com/locate/oceaneng

An exact analytic solution to the modified mild-slope equation for waves propagating over a trench with various shapes

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ARTICLE INFO

Article history:
Received 19 October 2011
Accepted 12 May 2012
Editor-in-Chief: A.I. Incecik
Available online 5 June 2012

Keywords:
Modified mild-slope equation
Exact analytic solution
Asymmetrical trench
Wave reflection
Mass-conserving matching condition
Total reflection

ABSTRACT

An exact analytic solution to the modified mild-slope equation (MMSE) in terms of Taylor series for waves propagating over an asymmetrical trench with various shapes is given. Because of the use of the MMSE, on one hand, the present analytic solution can be valid in the whole wave range from long waves to short waves, which is clearly superior to all previous long-wave analytic solutions; on the other hand, the present analytic solution can get rid of the limitation of the 'mild slope' assumption and be valid for bottom slope as high as 1:1. It is clarified that the improvement in solution accuracy by using the mass-conserving matching condition against the conventional matching condition mainly depends upon the jump quantities at all common boundaries. In addition, in comparison with previous approximate analytic model based on the approximate mild-slope equation, the present model is more accurate and can converge in the whole trench region without any restriction to trench depth. Based on the present MMSE solution, influence of trench dimensions to reflection effect is analyzed, which shows that total reflection effect increases when trench wall becomes steep and the phenomenon of zero reflection mainly occurs for symmetrical trenches.

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1. Introduction

It is well-known that the most popular wave equation in the linear wave regime is the mild-slope equation (MSE), which was originally proposed by Eckart (1951, 1952) and later improved by Berkhoff (1972, 1976) and re-derived by Smith and Sprinks (1975). It can also be found in Jonsson and Brink-Kjaer (1973) and Lozano and Meyer (1976).

However, the conventional MSE is only valid for bathymetries with their slopes being 'mild'. Using numerical solutions based on the hybrid finite element method, Tsay and Liu (1983) declared that the MSE can produce accurate results even for bottom slope being 1:1. Booij (1983) pointed out that Tsay and Liu's (1983) discovery is correct only for waves propagating parallel to the contours of the sloping bed, for waves propagating normal to the contours, the MSE produces accurate results only when the bottom slope is less than 1:3. Responding to the failure of the MSE to approximate adequately wave scattering by singly and doubly periodic ripple beds, Kirby (1986) derived an extended MSE which still includes the first order term related to ∇h only but differs from the conventional MSE. Further, Chamberlain and Porter (1995) derived the modified mild-slope equation (MMSE) in which both the bottom curvature term related to $\nabla^2 h$ and the slope-squared term related to $(\nabla h)^2$ are included. This MMSE was then rederived by Chandrasekera and Cheung (1997). Other improved versions to the MSE were given by Porter and Staziker (1995), Suh et al. (1997), Athanassoulis and Belibassakis (1999), Kim and Bai (2004) and Toledo and Agnon (2010).

However, seeking an analytic solution to the MSE or MMSE is extremely difficult though a few analytic solution in the long-wave limit has been constructed, see Liu and Lin (2005), Chang and Liou (2007), Jung et al. (2008) and Xie et al. (2011) in one-dimensional case. The main barrier comes from the fact that the linear dispersion relation is implicit with transcendental functions being included which leads to the implicity of coefficients in the MSE or MMSE. Recently, by using the real analytic implicit function theorem (Krants and Parks, 2002), Yang (2011) derived several crucial recursive formulae for calculating arbitrary order derivatives of two implicit coefficients in the MSE. Based on these recursive formulae, they can expand the two implicit coefficients k^2 and $\ln(CC_g)$ into Taylor series to arbitrary order for smooth topographies and successfully constructed exact analytic solutions to the MSE in terms of Taylor series for wave reflection by several one-dimensional piecewise smooth topographies. Very recently, by constructing several recursive formulae to calculate arbitrary order derivatives of the bottom curvature term and

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0029-8018/$ - see front matter © 2012 Elsevier Ltd. All rights reserved.
http://dx.doi.org/10.1016/j.oceaneng.2012.05.014

slope-square term, Liu et al. (2012) extended Yang's (2011) analytic technique to solve the MMSE (Chamberlain and Porter, 1995) for some one-dimensional piecewise smooth topographies.

In this paper, Liu et al.'s (2012) exact analytic technique for solving the MMSE is applied to conduct analytic studies to wave reflection by an asymmetrical trench with various shapes. A previous similar study to this wave problem was conducted by Jung et al. (2008) where an approximate analytic solution to the MSE by using Liu et al.'s (2004) approximate analytic technique based on Hunt's (1979) direct solution to the implicit wave dispersion was given. Intensive comparison between the present exact analytic model and Jung et al.'s (2008) approximate analytic model is made and the influence of trench dimensions to wave reflection effect is investigated based on the present model.

2. Recursive formulae

According to Chamberlain and Porter (1995), the MMSE can be expressed as

$$u_0\frac{d^2\eta}{dx^2}+\frac{du_0}{dx}\frac{d\eta}{dx}+(k^2u_0+u_1h''+u_2h'^2)\eta=0, \tag{1}$$

where

$$u_0=\frac{1}{2k}\tanh(kh)\left(1+\frac{2kh}{\sinh(2kh)}\right), \tag{2}$$

$$u_1=\frac{\operatorname{sech}^2(kh)}{4[2kh+\sinh(2kh)]}[\sinh(2kh)-2kh\cosh(2kh)], \tag{3}$$

$$\begin{aligned}u_2=&\frac{k\operatorname{sech}^2(kh)}{12[2kh+\sinh(2kh)]^3}[16(kh)^4+32(kh)^3\sinh(2kh)\\&-9\sinh(2kh)\sinh(4kh)+6kh(2kh+2\sinh(2kh))(\cosh^2(2kh)\\&-2\cosh(2kh)+3)],\end{aligned} \tag{4}$$

in which g is the gravitation acceleration, $h=h(x)$ is the water depth, and $k=k(x)$ is the local wave number which is determined by the following linear implicit dispersion relation:

$$\omega^2=gk\tanh kh. \tag{5}$$

It is noted that $u_0=CC_g/g$ with

$$C=\sqrt{\frac{g}{k}\tanh kh},\quad C_g=\frac{C}{2}\left(1+\frac{2kh}{\sinh 2kh}\right), \tag{6}$$

Eq. (1) can be equivalently rewritten as

$$\frac{d^2\eta}{dx^2}+\frac{d\ln(CC_g)}{dx}\frac{d\eta}{dx}+B(x)\eta=0 \tag{7}$$

with

$$B(x)=k^2+\frac{u_1g}{CC_g}h''+\frac{u_2g}{CC_g}h'^2. \tag{8}$$

Using the implicit dispersion relation equation (5), we have

$$CC_g=\frac{g\omega^2-\omega^4h+g^2k^2h}{2gk^2}, \tag{9}$$

$$u_1=\frac{(g^2k^2-\omega^4)(g\omega^2-g^2k^2h-\omega^4h)}{4g^2k^2(g^2k^2h-\omega^4h+g\omega^2)}, \tag{10}$$

$$\begin{aligned}u_2=&\frac{(g^2k^2-\omega^4)^4}{96g^2k^4(g^2k^2h-\omega^4h+g\omega^2)^3}\left[16(kh)^4+32(kh)^3\frac{2g\omega^2k}{g^2k^2}\right.\\&-\omega^4-72\frac{g^2\omega^4k^2(g^2k^2+\omega^4)}{(g^2k^2-\omega^4)^3}+12k^2h(g^2k^2h-\omega^4h+2g\omega^2)\\&\left.\times\frac{(g^2k^2+\omega^4)^2-2(g^4k^4-\omega^8)+3(g^2k^2-\omega^4)^2}{(g^2k^2-\omega^4)^3}\right],\end{aligned} \tag{11}$$

then

$$\frac{u_1g}{CC_g}h''=\frac{(g^2k^2-\omega^4)(g\omega^2-\omega^4h-g^2k^2h)}{2(g\omega^2-\omega^4h+g^2k^2h)^2}h''=\frac{B_1(x)}{2F^2(x)}, \tag{12}$$

$$\begin{aligned}\frac{u_2g}{CC_g}h'^2=&\frac{(g^2k^2-\omega^4)^4h'^2}{12(g\omega^2-\omega^4h+g^2k^2h)^4}\left[4k^2h^4+16kh^3\frac{g\omega^2k}{g^2k^2-\omega^4}\right.\\&-18\frac{g^2\omega^4(g^2k^2+\omega^4)}{(g^2k^2-\omega^4)^3}+3h(g^2k^2h-\omega^4h+2g\omega^2)\\&\left.\times\frac{(g^2k^2+\omega^4)^2-2(g^4k^4-\omega^8)+3(g^2k^2-\omega^4)^2}{(g^2k^2-\omega^4)^3}\right]=\frac{B_2(x)}{6F^4(x)},\end{aligned} \tag{13}$$

where

$$F(x)=g\omega^2-\omega^4h+g^2k^2h, \tag{14}$$

$$B_1(x)=(\omega^8h+\omega^2g^3k^2-g^4k^4h)h''-\omega^6gh'', \tag{15}$$

$$\begin{aligned}B_2(x)=&[2g^8h^4k^{10}-(8\omega^4g^6h^4-8\omega^2g^7h^3-3g^8h^2)k^8\\&+6(2\omega^8g^4h^4+\omega^2g^7h-4\omega^6g^5h^3-2\omega^4g^6h^2)k^6\\&-(8\omega^{12}g^2h^4-24\omega^{10}g^3h^3-24\omega^8g^4h^2+18\omega^6g^5h+9\omega^4g^6)k^4\\&+2(\omega^{16}h^4-4\omega^{14}gh^3-12\omega^{12}g^2h^2+15\omega^{10}g^3h)k^2\\&+9\omega^{16}h^2-18\omega^{14}gh+9\omega^{12}g^2]h'^2.\end{aligned} \tag{16}$$

Thus

$$B(x)=\frac{6k^2F^4(x)+3B_1(x)F^2(x)+B_2(x)}{6F^4(x)}. \tag{17}$$

Recently, several crucial recursion formulae for analytically calculating arbitrary order derivatives of the implicit coefficients in Eq. (7) were constructed by Yang (2011) and Liu et al. (2012) which are listed as follows, in which Recursion formulae 1 and 2 were given by Yang (2011) and Recursion formula 3 was given by Liu et al. (2012).

Recursion formula 1. Given a point ξ in the concerned physical domain, assume that $h(x)$ is analytic and positive in an interval $(\xi-\rho,\xi+\rho)$. Then the nth derivative of wavenumber function $k(x)$ in the interval can be recursively calculated as follows:

$$k'(x)=\frac{dk}{dx}=\frac{[\omega^4k(x)-g^2k^3(x)]h'(x)}{g\omega^2-\omega^4h(x)+g^2h(x)k^2(x)}=\frac{G(x)}{F(x)}, \tag{18}$$

$$k^{(n)}(x)=\frac{d^nk}{dx^n}=\frac{1}{F(x)}\left[G^{(n-1)}(x)-\sum_{i=0}^{n-2}\binom{n-1}{i}k^{(i+1)}(x)F^{(n-1-i)}(x)\right],\quad n=2,3,\ldots, \tag{19}$$

where $G^{(m)}(x)$ and $F^{(m)}(x)$ can be calculated recursively as follows:

$$(k^2(x))^{(m)}=\sum_{j=0}^{m}\binom{m}{j}k^{(j)}(x)k^{(m-j)}(x),\quad m=1,2,\ldots,n-1, \tag{20}$$

$$(k^3(x))^{(m)}=\sum_{j=0}^{m}\binom{m}{j}k^{(j)}(x)(k^2(x))^{(m-j)},\quad m=1,2,\ldots,n-1, \tag{21}$$

$$\begin{aligned}G^{(m)}(x)=&\sum_{j=0}^{m}\binom{m}{j}[\omega^4k^{(j)}(x)\\&-g^2(k^3(x))^{(j)}]h^{(m+1-j)}(x),\quad m=1,2,\ldots,n-1,\end{aligned} \tag{22}$$

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