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A nonlinear 7-DOF model for U-tanks of arbitrary shape

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ABSTRACT

This work presents a novel nonlinear 7-DOF model for ships equipped with U-tanks of arbitrary shape. The model uses the standard six degrees of freedom for the ship, in addition to a single degree of freedom for the tank fluid. The ship–tank interaction was modeled with Hamiltonian (analytical) mechanics, end external forces (such as those due to the surrounding ocean, actuator forces and various damping forces) were added later in a Newtonian framework. These external forces were not explicitly modeled in this work. The model was compared to two (significantly simpler and less powerful) models in the literature, one of which was experimentally verified. Under the same assumptions, the new model is identical to the experimentally verified one, and contains several effects not found in the other.

1. Introduction

The rolling motion of ships can be quite dangerous (Beck et al., 1989; Faltinsen, 1998; Fossen, 2011; Lloyd, 1998). As such, there is a necessity to reduce this unwanted motion. Unfortunately, the actuators used to move the ship are unsuitable to control roll (with the exception of rudder-roll-damping) (Fossen, 2011; Perez, 2005). Therefore, specialized control systems will have to be used.

There are several types of roll control devices. These include fins, gyro stabilizers, flume tanks and U-tanks (see Fig. 2) (Perez, 2005). These devices have intrinsic strengths and weaknesses. Fins are external and potentially vulnerable, increase drag, and the effectiveness is usually greater at high speeds. On the other hand, they have fast response times and are easy to control. U-tanks (also known as u-tube anti-roll tanks, u-shaped anti-roll tanks) are internal, and thus have no drag or vulnerability penalties, and are equally effective at low and high speeds. The downsides are that they are more complicated to model and control, can take up valuable space inside the hull and – most problematically – will have adverse effects in certain conditions.

A U-tank operates on the following principle: as the ship rolls, the fluid in the U-tank (usually water, but any liquid could be used) moves with it. For a passive tank, the fluid should move with the same frequency as the rolling motion, but lagging a quarter of a period behind (Lloyd, 1998). The vessel's kinetic energy is transformed into kinetic and potential energies of the tank fluid. Part of this energy is then dissipated by damping effects in the tank, such as vortex shedding and fluid viscous effects related to skin friction on the walls of the tank (Beck et al., 1989).

For a passive tank, both the ship and the tank fluid will move with the frequency of the waves (Fossen, 2011; Beck et al., 1989). This frequency will depend on the sea state, but the response of the ship will be much more severe if the exciting moment is at the ship's natural roll frequency (Nayfeh and Mook, 1995). Therefore, the U-tank should be constructed so that it is most effective at damping the roll motion at this frequency, and a U-tank should thus be designed to have its natural frequency approximately that of the ship's natural roll frequency (Faltinsen and Timokha, 2009; Lloyd, 1998).

The natural roll frequency will depend on loading conditions and other factors, and does not necessarily take the design value. The exact value might be unknown. The natural frequency of the U-tank fluid can be changed by changing its design or adding or removing fluid (Lloyd, 1998). Unfortunately, the natural frequency has very little sensitivity to changes in the fluid level, rendering it impractical to change the tank's natural frequency after it has been installed. Furthermore, a U-tank will have a limited range of exciting frequencies in which it is effective (Lloyd, 1998; Faltinsen and Timokha, 2009). For other frequencies, it may increase rather than decrease roll, and an incorrectly tuned U-tank can therefore cause more problems than it will solve. An incorrectly tuned U-tank may also simply have negligible effect on roll. Nevertheless, U-tanks are still in use throughout the world (Marzouk and Nayfeh, 2009; Moaleji and Greig, 2007; Perez, 2002, 2005).

The problems with a passive U-tank can be solved by adding an active control system. Pressurized air, pumps or simply a series of valves could be used to control the motion of the tank fluid.

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Ideally, a powerful and accurate model should be used to design and test such a system.

Several U-tank models exist in the literature. One of the oldest (Moaleji and Greig, 2007) is that of Goodrich (1968). Kagawa et al. (1989) presented a U-tank model for the purpose of reducing unwanted sway motion in skyscrapers. The most commonly used model is that of Lloyd (1989, 1998). More recently, U-tank models have appeared in Faltinsen and Timokha (2009), Marzouk and Nayfeh (2009), Neves et al. (2009), Holden et al. (2011) and Holden (2011).

The existing models have several limitations. Chief among these are that they are derived for rectangular-prism U-tanks (i.e., U-tanks consisting of three rectangular boxes) (Holden et al., 2011; Kagawa et al., 1989; Lloyd, 1989, 1998; Marzouk and Nayfeh, 2009; Moaleji and Greig, 2007; Neves et al., 2009; Sellars and Martin, 1992),[1] despite the fact that several actually installed tanks do not match this shape (Perez, 2002, 2005; Sellars and Martin, 1992). A model valid also for more generic shapes is therefore likely to be useful. Furthermore, most existing models are linear (Frahm, 1911; Goodrich, 1968; Kagawa et al., 1989; Lloyd, 1989, 1998; Moaleji and Greig, 2007), and technically only valid for low-amplitude motions. Finally, they are often limited in degrees of freedom, typically to two (usually roll and a tank state) (Frahm, 1911; Goodrich, 1968; Holden et al., 2011; Kagawa et al., 1989) or four (usually sway, roll, yaw and a tank state) (Faltinsen and Timokha, 2009; Lloyd, 1989, 1998).

It is known that the modes most affected by/affecting a U-tank are sway, roll and yaw, and that a 2-DOF model might be too low order for a thorough analysis (Faltinsen and Timokha, 2009; Lloyd, 1998). However, all degrees of freedom will be affected by and affect a U-tank, and a high-order model can always easily be reduced to a low-order model, but not the other way around.

In this work, we therefore present a novel 7-DOF nonlinear model for ships equipped with a U-tank of arbitrary shape. The ship–tank interaction is modeled with Hamiltonian (analytical) mechanics,[2] and the forces and moments of the surrounding ocean added later in a Newtonian framework (these latter forces are not explicitly modeled in this work). The novel model can accurately model U-tanks of arbitrary shape, describe high-amplitude motion and has seven degrees of freedom (the well-known six of the ship and a single tank state). The main disadvantage of the new model is that it is quite complex. Simplifications can, however, be easily made. Furthermore, the motion of the tank fluid is assumed to be one-dimensional and as a consequence does not accurately model the behavior of the surface motion of the tank fluid.

The model is compared to the commonly used (linear, 4-DOF, rectangular-prism U-tank) model of Lloyd (1989, 1998) and the experimentally verified (nonlinear, 2-DOF, rectangular-prism U-tank) model of Holden et al. (2011). Under the same assumptions as the existing models, the novel model is auxiliary perfect match for that of Holden et al. (2011), and incorporates several effects left out of the model of Lloyd (1989, 1998).

The rest of the paper is organized as follows: Section 2 presents a brief nomenclature. Section 3 defines the reference frames and kinematic representation used in this work. Section 4 presents the main assumptions that form the basis of the model derivation, and properly defines the U-tank and the state which describes the motion of the U-tank fluid. Section 5 presents a Hamiltonian model of the ship–tank system (excluding the effects of the surrounding ocean). Section 6 adds the effects of forces not presented in Section 5, such as those of the surrounding ocean, actuators and dissipative effects. Section 7 compares the novel model to those of Lloyd (1989, 1998) and Holden et al. (2011). Section 8 presents the conclusions. In the appendices are found auxiliary results and derivations, to wit: derivation of the potential energy (Appendix A), derivation of the kinetic energy (Appendix B), properties of certain matrices needed in the proofs (Appendix C), a definition of a virtual work principle used in deriving the dynamics (Appendix D) and the derivation of the Hamiltonian dynamics (Appendix E). The references are found at the very end of this paper.

2. Nomenclature

This section lists the variables used in this work.

In general, matrices will be written in uppercase with italic typeface, e.g., A. Vectors and scalars are typically written in lowercase with italic typeface, e.g., a. Whether it is a vector or scalar will be stated in the text, but should largely be clear from context.

If the vector has an interpretation as a point, velocity or angular velocity in physical $\mathbb{R}^3$, a superscript will typically denote which reference frame is used to describe the vector, e.g., r^n would denote that r is given in the n-frame. Only two frames are used, the b-frame (fixed to the ship) and the n-frame (fixed to the Earth and considered inertial), see Section 3.

$\mathbb{I}_n \in \mathbb{R}^{n\times n}$: the n-by-n identity matrix.
$0_{m\times n} \in \mathbb{R}^{m\times n}$: the m-by-n zero matrix.
$e_z = [0,0,1]^\top$: unit vector in the z-direction (in $\mathbb{R}^3$).
$S(\cdot) \in SS(3) \subset \mathbb{R}^{3\times 3}$: a skew-symmetric matrix representing the cross-product in $\mathbb{R}^3$. $S(x)y = x\times y \; \forall x,y \in \mathbb{R}^3$.
$R \in SO(3) \subset \mathbb{R}^{3\times 3}$: rotation matrix representing the orientation of the b-frame relative to the n-frame. If r is a vector in physical $\mathbb{R}^3$, then $r^n = Rr^b$.
$g > 0 \in \mathbb{R}$: the acceleration of gravity.
$x^n = [x,y,z]^\top \in \mathbb{R}^3$: the position of the origin of the b-frame, described in the n-frame.
$\eta = [\eta_r, \eta_i^\top]^\top \in \mathbb{R}^4$: quaternion describing the orientation of the b-frame relative to the n-frame. $\eta_r = \mathrm{Re}(\eta) \in \mathbb{R}$, $\eta_i = \mathrm{Im}(\eta) \in \mathbb{R}^3$.
$q_t \in \mathbb{R}$: generalized position of the tank fluid.
$q = [x^{n\,\top}, \eta^\top, q_t]^\top \in \mathbb{R}^8$: generalized position.
$\nu^b \in \mathbb{R}^3$: the velocity of the b-frame relative to the n-frame, described in the b-frame.
$\omega^b \in \mathbb{R}^3$: the angular velocity of the b-frame relative to the n-frame, described in the b-frame.
$\nu = [\nu^{n\top}, \omega^{b\top}, \dot{q}_t]^\top \in \mathbb{R}^7$: generalized velocity.
$G(\eta) \in \mathbb{R}^{3\times 4}$: matrix relating $\dot{\eta}$ to ω^b; $\dot{\eta} = \frac{1}{2}G^\top(\eta)\omega^b$.
$\mathcal{P}(\eta) \in \mathbb{R}^{7\times 8}$: matrix relating $\dot{q}$ to ν; $\dot{q} = \mathcal{P}^\top(\eta)\nu$.
ϕ, θ, ψ: roll–pitch–yaw Euler angles.
$m > 0 \in \mathbb{R}$: mass of the ship.
$I = I^\top > 0 \in \mathbb{R}^{3\times 3}$: the ship's moment of inertia tensor, in the body frame.
$r_g^b = [x_g^b, 0, z_g^b]^\top$: position of the ship's center of gravity (excluding tank fluid), in the b-frame.
$\sigma \in \mathbb{R}$: parameter describing the geometry of the tank.
$r_t^b(\sigma) = [x_t^b, y_t^b(\sigma), z_t^b(\sigma)]^\top \in \mathbb{R}^3$: a function describing the centerline of the U-tank, in the b-frame.
$A(\sigma) > 0 \in \mathbb{R}$: cross-sectional area of the tank.
$\rho_t > 0 \in \mathbb{R}$: density of the tank fluid.
$\varsigma_0 > 0 \in \mathbb{R}$: mean level of tank fluid.

[1] Technically, the model in Marzouk and Nayfeh (2009) is for a tank consisting of three cylinders, but this is functionally identical to a rectangular-prism U-tank.

[2] Most other models have been derived by Newtonian mechanics (Faltinsen and Timokha, 2009; Frahm, 1911; Goodrich, 1968; Lloyd, 1989, 1998; Moaleji and Greig, 2007; Marzouk and Nayfeh, 2009; Neves et al., 2009; Sellars and Martin, 1992), but both approaches result in the same dynamic equations under the same assumptions.