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Bottom friction and erosion beneath long-crested and short-crested nonlinear random waves

Dag Myrhaug *, Lars Erik Holmedal

Department of Marine Technology, Norwegian University of Science and Technology, NO-7491 Trondheim, Norway

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ABSTRACT

A practical approach for calculating the bottom shear stress beneath long-crested (2D) and short-crested (3D) nonlinear random waves is provided. The approach is based on assuming the waves to be a stationary narrow-band random process and by adopting the Forristall (2000) wave crest height distribution representing both 2D and 3D nonlinear random waves. Results are presented for laminar, smooth turbulent and rough turbulent flow. Examples are also included to illustrate the applicability of the results for practical purposes using data typical for field conditions; the mobile layer thickness in sheet flow representing rough turbulent flow; erosion of mud representing smooth turbulent flow and deposition of mud representing laminar flow.

1. Introduction

Ocean surface waves show a complex three-dimensional irregular pattern where the sharpening of the wave crests manifests wave nonlinearity. In finite water depths this affects the bottom wave boundary layer, which is a thin flow region at the seabed dominated by friction arising from the bottom roughness. The wave boundary layer flow determines the bottom shear stress, which affects many phenomena in coastal engineering, e.g. sediment transport and assessment of the stability of scour protection in the marine environment. A review is, e.g., given in Holmedal et al. (2003).

For the prediction of the bottom friction under nonlinear random waves, a commonly used procedure is to substitute the wave height, H, or the bed orbital velocity amplitude, U, with their *rms* (root-mean-square) values H_{rms} and U_{rms}, respectively, in an otherwise deterministic approach, see e.g. Soulsby (1997). However, this procedure does not account for the stochastic feature of the processes included.

Other studies of bottom friction for long-crested random waves have been made by, e.g., Madsen (1994), Simons et al. (1994, 1996), Myrhaug (1995), Myrhaug et al. (1998, 2001), Holmedal et al. (2000, 2003), and Myrhaug and Holmedal (2002).

The purpose of this study is to provide a practical approach for calculating the bottom shear stress beneath long-crested (2D) and short-crested (3D) nonlinear random waves. The approach is based on assuming the waves to be a stationary narrow-band random process and adopting the Forristall (2000) wave crest height distribution representing both 2D and 3D random waves. The approach does not give any information about the boundary layer flow itself, but the maximum bottom shear stress can be estimated to a degree of accuracy suitable for many practical purposes. The cumulative distribution function of individual bed shear stress maxima for 2D and 3D nonlinear random waves is determined. Results are presented for laminar, smooth turbulent and rough turbulent flow. The maximum bottom shear stress is the quantity of interest when, e.g., assessing sediment mobility at the seabed.

2. Theoretical background

The maximum bottom shear stress under the wave crest of a single wave in a sea state with stationary narrow-band random waves, τ_m, is given as

$$\frac{\tau_m}{\rho} = \frac{1}{2} f_w U_c^2 \tag{1}$$

where U_c is the maximum near-bed orbital velocity under the wave crest, ρ is the density of the fluid and f_w is the wave friction factor given as for laminar (Eq. (2)), smooth turbulent (Eq. (4)) and rough turbulent flow (Eqs. (6)–(9)).

For laminar flow, the wave friction factor is given as that for Stokes' second problem (Schlichting, 1979)

$$f_w = 2Re^{-0.5} \quad \text{for} \quad Re \leq 3 \times 10^5 \tag{2}$$

* Corresponding author.
E-mail address: dag.myrhaug@ntnu.no (D. Myrhaug).

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where

$$Re = \frac{U_c A_c}{\nu} \tag{3}$$

is the Reynolds number associated with the wave motion, $A_c = U_c/\omega_p$ is the maximum near-bed orbital displacement under the wave crest, ω_p is the spectral peak wave frequency and ν is the kinematic viscosity of the fluid.

For smooth turbulent flow, the Myrhaug (1995) smooth bed wave friction factor is adopted

$$f_w = rRe^{-s} \text{ for } \quad Re > 3\times 10^5 \tag{4}$$

with the coefficients

$$(r,s) = (0.0450, 0.175) \tag{5}$$

Alternative coefficients (r, s) for smooth turbulent flow are given in Soulsby (1997).

For rough turbulent flow, the friction factor proposed by Myrhaug et al. (2001) is used

$$f_w = c\left(\frac{A_c}{z_0}\right)^{-d} \tag{6}$$

$$(c,d) = (18,1) \quad \text{for} \quad 20 \lesssim A_c/z_0 \lesssim 200 \tag{7}$$

$$(c,d) = (1.39, 0.52) \quad \text{for} \quad 200 \lesssim A_c/z_0 \lesssim 11000 \tag{8}$$

$$(c,d) = (0.112, 0.25) \quad \text{for} \quad 11000 \lesssim A_c/z_0 \tag{9}$$

where $z_0 = 2.5d_{50}/30$ is the bed roughness based on the median grain size diameter d_{50}. Note that Eq. (8) corresponds to the coefficients given by Soulsby (1997) obtained as best fit to data for $10 \lessapprox A_c/z_0 \lessapprox 10^5$. The advantage of using this friction factor for rough turbulent flow is that it is possible to derive the stochastic approach analytically.

At a fixed point in a sea state with stationary narrow-band random waves consistent with the Stokes second order regular waves in finite water depth h the non-dimensional nonlinear crest height, $w_c = \eta_c/a_{rms}$, and the non-dimensional nonlinear maximum horizontal particle velocity evaluated at the seabed, $\hat{U}_c = U_c/U_{rms}$, are (Dean and Dalrymple, 1984)

$$w_c = \hat{a} + O(k_p a_{rms}) \tag{10}$$

$$\hat{U}_c = \hat{a} + O(k_p a_{rms}) \tag{11}$$

Here $\hat{a} = a/a_{rms}$ is the non-dimensional linear wave amplitude, where the linear wave amplitude a is made dimensionless with the *rms* value a_{rms}, and

$$U_{rms} = \frac{\omega_p a_{rms}}{\sinh k_p h} \tag{12}$$

Moreover, $O(k_p a_{rms})$ denotes the second order (nonlinear) terms, which are proportional to the characteristic wave steepness of the sea state, $k_p a_{rms}$, where k_p is the wave number corresponding to ω_p(=peak frequency of wave spectrum) given by the dispersion relationship for linear waves (which is also valid for the Stokes second order waves)

$$\omega_p^2 = g k_p \tanh k_p h \tag{13}$$

and g is the acceleration of gravity.

Now Eq. (10) can be inverted to give $\hat{a} = w_c - O(k_p a_{rms})$, which substituted in Eq. (11) gives $\hat{U}_c = w_c + O(k_p a_{rms})$. Thus it appears that $\hat{a}$ can be replaced by w_c in the linear term of $\hat{U}_c$, because the error involved is of second order. Consequently, by neglecting terms of $O(k_p a_{rms})$ the maximum near-bed orbital velocity under the wave crest in dimensional form can be taken as

$$U_c = \frac{\omega_p \eta_c}{\sinh k_p h} \tag{14}$$

By substituting Eq. (14) in Eqs. (1)–(4) and using that $A_c = U_c/\omega_p$, the non-dimensional maximum shear stress under the wave crest, $\tau_c = \tau_m/\tau_{rms}$, for laminar ($r=2$, $s=0.5$) and smooth turbulent flow is given as

$$\tau_c = w_c^{2-2s} \tag{15}$$

where

$$\frac{\tau_{rms}}{\rho} = \frac{1}{2} rRe_{rms}^{-s} U_{rms}^2; \quad Re_{rms} = \frac{U_{rms} A_{rms}}{\nu} \tag{16}$$

and $A_{rms} = U_{rms}/\omega_p$.

Similarly, by substituting Eq. (14) in Eqs. (1) and (6) the non-dimensional maximum shear stress under the wave crest for rough turbulent flow is given as

$$\tau_c = w_c^{2-d} \tag{17}$$

where

$$\frac{\tau_{rms}}{\rho} = \frac{1}{2} c\left(\frac{A_{rms}}{z_0}\right)^{-d} U_{rms}^2 \tag{18}$$

Now the Forristall (2000) parametric crest height distribution based on simulations using second order theory is adopted. The simulations were based on the Sharma and Dean (1981) theory; this model includes both sum-frequency and difference-frequency effects. The simulations were made both for 2D and 3D random waves. A two-parameter Weibull distribution with the cumulative distribution function (*cdf*) of the form

$$P(w_c) = 1 - \exp\left[-\left(\frac{w_c}{\sqrt{8}\alpha}\right)^{\beta}\right]; \quad w_c \ge 0 \tag{19}$$

was fitted to the simulated wave data. The Weibull parameters α and β were estimated from the fit to the simulated wave data, and are based on the wave steepness S_1 and the Ursell parameter U_R defined by

$$S_1 = \frac{2\pi}{g}\frac{H_s}{T_1^2} \tag{20}$$

and

$$U_R = \frac{H_s}{k_1^2 h^3} \tag{21}$$

Here H_s is the significant wave height, T_1 is the spectral mean wave period and k_1 is the wave number corresponding to T_1. The wave steepness and the Ursell number characterize the degree of nonlinearity of the waves in finite water depth. At zero steepness and zero Ursell number the fits were forced to match the Rayleigh distribution, i.e. $\alpha = 1/\sqrt{8} \approx 0.3536$ and $\beta = 2$. Note that this is the case for both 2D and 3D linear waves. The resulting parameters for the 2D-model are

$$\alpha_{2D} = 0.3536 + 0.2892 S_1 + 0.1060 U_R$$
$$\beta_{2D} = 2 - 2.1597 S_1 + 0.0968 U_R^2 \tag{22}$$

and for the 3D-model

$$\alpha_{3D} = 0.3536 + 0.2568 S_1 + 0.0800 U_R$$
$$\beta_{3D} = 2 - 1.7912 S_1 - 0.5302 U_R + 0.284 U_R^2 \tag{23}$$

Forristall (2000) demonstrated that the wave setdown effects were smaller for short-crested than for long-crested waves, which is due to that the second-order negative difference-frequency terms are smaller for 3D waves than for 2D waves. Consequently the wave crest heights are larger for 3D waves than for 2D waves.

Based on the Forristall distribution the *cdf* of τ_c is obtained by transformation of random variables, i.e. using Eqs. (15), (17) and (19) it follows that τ_c is given by the two-parameter Weibull