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Extreme wave and wind response predictions

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ABSTRACT

The aim of the paper is to advocate effective stochastic procedures, based on the First Order Reliability Method (FORM) and Monte Carlo simulations (MCS), for extreme value predictions related to wave and wind-induced loads.

Due to the efficient optimization procedures implemented in standard FORM codes and the short duration of the time domain simulations needed (typically 60–300 s to cover the hydro- and aerodynamic memory effects in the response) the calculation of the mean out-crossing rates of a given response is fast. Thus non-linear effects can be included. Furthermore, the FORM analysis also identifies the most probable wave episodes leading to given responses.

Because of the linearization of the failure surface in the FORM procedure the results are only asymptotically exact and thus MCS often also needs to be performed. In the present paper a scaling property inherent in the FORM procedure is investigated for use in MCS in order to reduce the necessary simulation time. Thereby uniform accuracy for all exceedance levels can be achieved by a modest computational effort, even for complex non-linear models. As an example extreme responses of a floating offshore wind turbine are analyzed taking into consideration both stochastic wave and wind-induced loads.

1. Introduction

The First Order Reliability Method (FORM) has been used widely within structural reliability analyses. However, it is also an efficient method for extreme value predictions, as suggested by Der Kiureghian (2000). Due to the efficient optimization procedures implemented in standard FORM codes and the short duration of the time domain simulations needed (typically 60–300 s to cover the hydro- and aerodynamic memory effects in the response) the calculation of the out-crossing rate of the response is very fast. Thus complicated non-linear effects can be included.

Three different applications are summarized in Jensen (2007a). The first dealt with a jack-up rig where the deck surge is considered taking into account second-order stochastic waves. This is usually not possible due to excessive computational time, but the efficiency of the FORM analysis makes it possible. The second-order wave elevation is important as it increases the crest height and thereby the overturning moment.

The second application was the roll motion of ships. It was shown that even bifurcation type of responses like parametric rolling can be dealt with by FORM. Finally, the horizontal motion of a Tension Leg Platform (TLP) for an offshore wind turbine was analyzed taking into account large horizontal motions and the elastic deformation of the tower. The heave and pitch motions were disregarded and the tendons considered inextensible. This study was later extended to include flexible tendons, Joensen et al. (2007). In both studies, the wind load is taken as a constant value.

The reliability index calculated by the FORM analysis is strictly inversely proportional to the square root of the intensity of the load spectrum, irrespective of the non-linearity in the system; see e.g. Jensen (2007b), Garrè and Der Kiureghian (2010) and Jensen (2011), provided the relative frequency distribution of the load spectrum is unchanged. The physics behind this dependence is that within the FORM approximation the same load scenario leading to the specified response is obtained irrespectively of the intensity of the load. Only the probability that this load scenario occurs changes with the intensity. The scaling property also holds if e.g. a control algorithm is present as long as the control algorithm depends solely on the wave on wind-induced responses.

For low probabilities of exceedance the FORM is significantly faster than direct Monte Carlo simulations and usually very accurate as shown in the examples summarized in Jensen (2007a). However, at probability levels relevant for design the FORM analysis only provides an approximation. Exact results can be obtained by Monte Carlo simulations, but the necessary length

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of the time domain simulations for low out-crossing rates might be prohibitively long.

In such cases the scaling property mentioned above makes it possible to increase the out-crossing rates and thus reduce the necessary length of the time domain simulations by applying a larger load intensity than relevant from a design point of view. The reliability index and the corresponding mean out-crossing rate thus obtained can afterwards be scaled down to the actual intensity of the load. This will make possible complete long-term extreme response calculations taking into account the wave and wind distribution at the location in question. The accuracy of this approach will be investigated in the present paper, considering the response of an offshore TLP wind turbine.

The procedure is an alternative to standard Monte Carlo simulations, where the actual load intensity is applied, but reduced threshold levels are considered, followed by a Weibull or Gumbel distribution fit in order to get the probability of exceedance of the desired threshold level. The standard procedure has the disadvantage that the accuracy of this fit decreases with the non-linearity of the system, which is not the case with the present procedure.

The paper will start with an outline of the FORM, emphasizing its application to extreme wave and wind load predictions.

2. The First Order Reliability Method applied to combined wave and wind loads

The excitation or input process is considered to be a stationary stochastic process. The input process is the wave elevation and the associated wave kinematics together with the stochastic wind velocity field.

For moderate sea states the wave elevation can be considered as Gaussian distributed, whereas for more severe sea states non-linearities in the wave model must be incorporated. A second-order stochastic wave model was used in Jensen and Capul (2006) and could easily be included here, too. However, for brevity, linear, long-crested waves are assumed in this paper. Hence, the normal distributed wave elevation $H(x,t)$ as a function of space x and time t can be written in discretized form as

$$H(x,t) = \sum_{i=1}^{m} (u_i c_i(x,t) + \bar{u}_i \bar{c}_i(x,t)) \tag{2.1}$$

where the variables $u_i, \bar{u}_i$ are statistically independent, standard normal distributed variables to be determined by the stochastic procedure and with the deterministic coefficients given by

$$\begin{aligned} c_i(x,t) &= \sigma_i \cos(\omega_i t - k_i x) \\ \bar{c}_i(x,t) &= -\sigma_i \sin(\omega_i t - k_i x) \\ \sigma_i^2 &= S_{wave}(\omega_i) d\omega_i \end{aligned} \tag{2.2}$$

where ω_i, $k_i = \omega_i^2/g$ are the m discrete frequencies and wave numbers applied. Here g is the acceleration of gravity. Furthermore, $S_{wave}(\omega)$ is the wave spectrum and $d\omega_i$ the increment between the m discrete frequencies applied.

Similarly, the stochastic part, $V(t)$, of the wind velocity around the mean wind speed U is modeled as normal distributed and statistically independent of the waves. The stochastic fluctuation $V(t)$ is specified by a wind spectrum $S_{wind}(\omega)$ and written

$$\begin{aligned} V(t) &= \sum_{j=m+1}^{n} \sigma_{wi}(u_j \cos(\omega_i t) - \bar{u}_j \sin(\omega_i t)) \\ \sigma_{wj}^2 &= S_{wind}(\omega_j) d\omega_j \end{aligned} \tag{2.3}$$

where $d\omega_j$ is the increment between the n–m discrete frequencies used for the wind spectrum. The spatial coherence is ignored in the example to follow, but have been included.

From the wave and wind models, Eqs. (2.1)–(2.3), and the associated kinematics any non-linear combined wave and wind-induced response $\phi(t)$ can in principle be determined by a time domain analysis using a proper hydro- and aerodynamic model:

$$\phi = \phi(t \mid u_1, \bar{u}_1, u_2, \bar{u}_2, \ldots, u_n, \bar{u}_n) \tag{2.4}$$

Each of these realizations represents the response for a possible wave and wind scenario. The realization which exceeds a given threshold ϕ_0 at time $t=t_0$ with the highest probability is sought. This problem can be formulated as a limit state problem, well-known within time-invariant reliability theory, Der Kiureghian (2000, Eq. (14b))

$$G(u_1, \bar{u}_1, u_2, \bar{u}_2, \ldots, u_n, \bar{u}_n) \equiv \phi_0 - \phi(t_0 \mid u_1, \bar{u}_1, u_2, \bar{u}_2, \ldots, u_n, \bar{u}_n) = 0 \tag{2.5}$$

An approximate solution can be obtained by use of First Order Reliability Methods (FORM). The limit state surface G is given in terms of the statistically independent standard normal distributed variables $\{u_i, \bar{u}_i\}$ and determination of the design point, $\{u_i^*, \bar{u}_i^*\}$, defined as the point on the failure surface, $G=0$, with the shortest distance to the origin, is rather straightforward, Der Kiureghian (2000). A linearization around this point replaces Eq. (2.5) with a hyper-plane in $2n$ space. The distance, β_{FORM}, from the hyper-plane to the origin is denoted the (FORM) reliability index and is equal to

$$\beta_{FORM} = -\Phi^{-1}(\phi(t_0 \mid u_1, \bar{u}_1, u_2, \bar{u}_2, \ldots, u_n, \bar{u}_n) > \phi_0 \mid \text{FORM linearization}) \tag{2.6}$$

where Φ^{-1} is the inverse of the standard normal distribution function. Thus the reliability index directly relates to the probability of exceedance The calculation of the design point $\{u_i^*, \bar{u}_i^*\}$ and the associated value of β_{FORM} can be performed by standard reliability codes, e.g. Det Norske Veritas (2002), or by standard optimization codes (i.e. minimizing the distance β with the constraint Eq. (2.5)) in which $\phi(t_0)$ has to be calculated by numerical integration for a number of combinations of $\{u_i, \bar{u}_i\}$ until the design point is reached. The integration must cover a sufficiently long time period $\{0, t_0\}$ to avoid any influence on $\phi(t_0)$ of the initial conditions at $t=0$, i.e. to be longer than the memory in the system. As no explicit expression for $\phi(t_0)$ is needed, any kind of non-linearities can be incorporated. Note that the reliability procedure only gets $\phi(t_0)$ as input and no information about the time evolution leading to $\phi(t_0)$.

Typical values of t_0 would be 1–2 min, depending on the damping in the system. Hence, to avoid repetition in the wave system and for representation of typical wave spectra $m=10$–50 would at least be needed, whereas for wind spectra $n-m=5$–10 frequencies are considered sufficient for the present illustrative example. A more realistic discretization of the wave and wind spectra is applied in Perdrizet and Averbuch (2011).

3. Mean out-crossing rates and peak value distribution

The time-invariant peak distribution of the responses is of main interest for design. Extreme values in stationary stochastic processes are usually dealt with either by the Poisson model or order statistics. In both cases the basic parameter is the mean out-crossing rate. For upward unbounded responses, both models lead to the Gumbel distribution for the asymptotic extreme response:

$$\begin{aligned} &\text{Order statistics} \to \text{Poisson up-crossings}: \\ &P(\max(R) < \phi_0 \mid 0 < t < T) = F_p^N(\phi_0) \\ &\quad = \left(1 - \frac{\nu(\phi_0)}{\nu(0)}\right)^N = \left(1 - \frac{T\nu(\phi_0)}{N}\right)^N \underset{N\to\infty}{\to} e^{-\nu(\phi_0)T} \end{aligned} \tag{3.1}$$

where T and $N=\nu(0)T$ are the total time and number of peaks, respectively, and $F_p(\phi_0)$ the distribution function of the peak of ϕ_0. Generally, the Poisson model is accurate for high threshold