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Comparison of theoretical and experimental wall pressure wavenumber–frequency spectra for axisymmetric and flat-plate turbulent boundary layers

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ABSTRACT

The measurement and analysis of turbulent boundary layer wall pressure fluctuations using a wavenumber filter of sensors provide quantitative knowledge of turbulence physics. In addition, the sources of flow-induced noise and vibration for towed SONAR arrays can be determined. An axisymmetric turbulent boundary layer can have significantly different features than those of a comparable flat-plate boundary layer. Here, a detailed comparison of the distribution of wall pressure energy in both wavenumber and frequency between flat-plate and thick axisymmetric boundary layers is presented. The background theory of wavenumber–frequency spectra and state-of-the-art models for flat-plate boundary layers are discussed. The widely used model of Chase (1987), valid for flat-plate boundary layers over a wide range of Reynolds numbers, is used and combined with a sensor response function to allow the effects of spatial averaging to be considered. It is demonstrated that when measured boundary layer parameters for the axisymmetric case are used in the Chase flat-plate model, the results accurately predict the axisymmetric boundary layer wall pressure measurements.

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1. Introduction

Turbulence is a source of background noise which limits the performance of hull-mounted and towed SONAR arrays. It occurs in moderate to high Reynolds number boundary layer flows and exists over a broad range of length and time scales. Turbulent energy exerts wall pressure fluctuations (or flow noise) over the boundary and induces structural vibration of the boundary itself. Identification and characterization of these dominant noise sources will allow for improvements in array and sensor design, as well as scientific investigations of turbulent boundary layer physics. Wavenumber–frequency spectra, analytical models for wall pressure, and an understanding of boundary layer geometry are all key elements in addressing the flow noise problem.

1.1. Wavenumber–frequency spectra

Wavenumber–frequency (or '$k-\omega$') spectra allow the relative intensity of sources of flow noise energy to be identified. The frequency range of the measurement allows direct evaluation of the wall pressure fluctuations which occur at higher frequencies and the flow-induced structural vibrations present at lower frequencies. Propagation speeds related to convected turbulence, structural waves, and acoustic energy are also easily determined.

An example of a wavenumber–frequency plot of turbulent energy spectra is given in Fig. 1. The wavespeed associated with any of the energy displayed on a wavenumber–frequency plot is defined by its relative location within the plot. The 'acoustic cone' (not to scale) is the region on a wavenumber–frequency plot where acoustic energy lies. The slopes of the limiting edges of this region are defined by the speed of sound. Turbulent energy is distributed across the 'convective ridge,' the slope ($d\omega/dk$) of which is defined by the mean convection velocity of the turbulent structures in the flow. In this example, the energy in the convective ridge has been aliased. Analogous to temporal aliasing beyond the Nyquist frequency ω_{Nyquist}, spatial aliasing occurs when energy is sampled at discrete points at spacing Δx. This results in energy appearing to wrap around one edge of the plot, defined by the Nyquist wavenumber $k_{\text{Nyquist}}=\pi/\Delta x$, to the other.

A cut across a wavenumber–frequency plot at a discrete frequency reveals the distribution of energy and the shape of the convective ridge for this frequency. In the example shown in Fig. 2, taken at ω_o as indicated on Fig. 1, the convective ridge is apparent, and there is no aliased energy. The 'low wavenumber region' indicates where turbulent energy has bled into the acoustic cone. It is this turbulent energy which contaminates the beamformed outputs of acoustic arrays.

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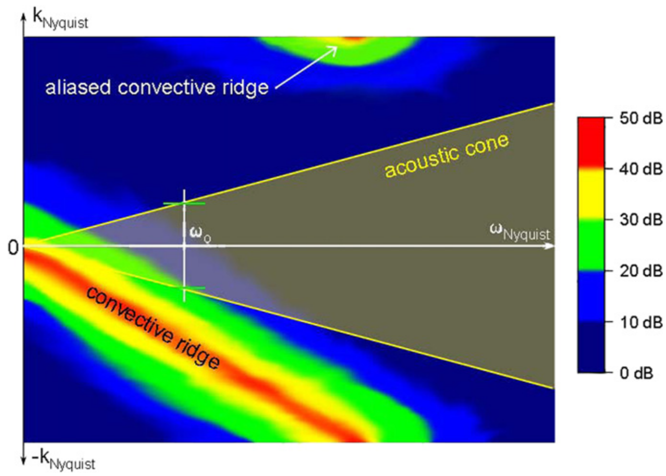

Fig. 1. Example of a wavenumber–frequency ($k-\omega$) plot of turbulent energy. The given color intensity scale is arbitrary. Apparent features include the convective ridge and aliased convective energy. The acoustic cone (not to scale) is the area of a $k-\omega$ plot where acoustic energy lies, the boundaries of which are defined by the speed of sound. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

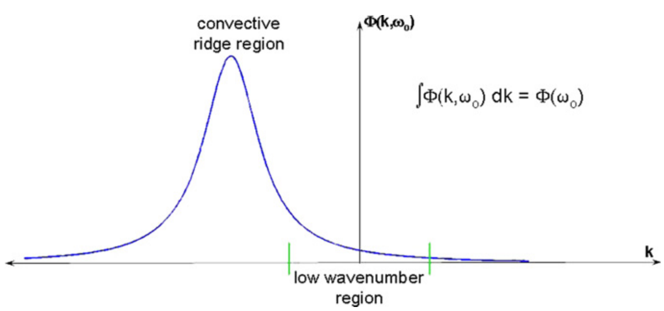

Fig. 2. Example of a 'cut' of a wavenumber–frequency ($k-\omega$) spectra plot of turbulent energy taken at frequency ω_o, as indicated on Fig. 1. The integral of this plot provides the autospectral level of the energy at this frequency.

The autospectra of turbulent energy are related to the wavenumber–frequency spectra through the integral relation:

$$\Phi(\omega) = \int_{-\infty}^{\infty} \Phi(k,\omega)\,\mathrm{d}k. \tag{1}$$

While the wavenumber–frequency spectra allow energy from acoustic and non-acoustic sources to be resolved as a function of wavenumber and frequency, the autospectra provide a measure of the total energy from all sources as a function of frequency.

A direct measurement of wavenumber–frequency spectra involves simultaneous acquisition of the time series from equally spaced pressure sensors which form a sensor wavenumber filter (Fig. 3). After the time series are low-pass filtered in frequency and digitized, a fast Fourier transform (FFT) is applied to finite samples of each time series. Multiple samples are then averaged together to produce the spectra. The number of sensors combined with the sensor spacing determines the array aperture which controls the width of the wavenumber bins. Turbulent wall pressure spectra typically do not contain narrow band features such that the aperture of the wavenumber filter is small relative to that of an acoustic sensor array. Temporal sampling rate determines the Nyquist frequency of the measurement, and the sensor spacing Δx determines the Nyquist wavenumber k_{Nyquist} and the range over which unaliased estimates are obtained. Although the problem of temporal aliasing can be effectively solved with low-pass filtering, the problem of spatial aliasing is more difficult, and must be considered when analyzing measured spectra. The size of the sensors leads to spatial averaging which effectively filters energy at higher wavenumbers and passes energy at lower wavenumbers. This mechanism can be used as a low-pass spatial filter and as a means of controlling spatial aliasing. In practice, the size of the pressure sensor and capability of the data acquisition system constrain the design of the wavenumber filter. Furthermore, for hydrodynamic applications, the sensors must be sufficiently rugged to withstand dynamic fluid loading and hydrostatic pressure.

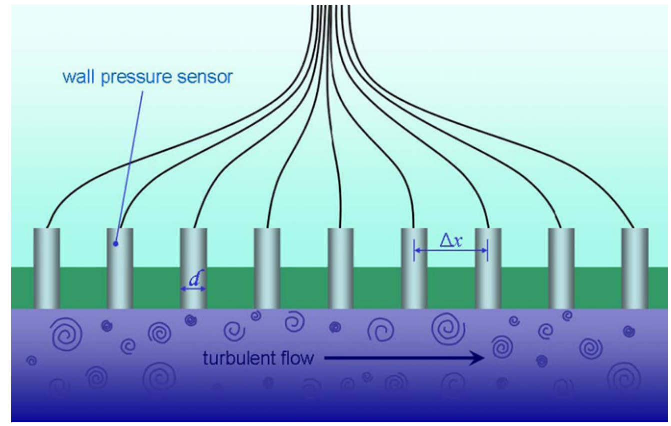

Fig. 3. Schematic of an experimental wall pressure sensor wavenumber filter. Wall pressure sensors of diameter d are separated at a spacing Δx. Measurements are taken at the fluid–solid interface underneath a turbulent flow.

1.2. Analytical models for the wall pressure spectrum in a flat-plate boundary layer

Corcos (1963) proposed a model for the cross-spectra of wall pressure fluctuations based on the measurements of Willmarth and Wooldridge (1962). The similarity scaling Corcos used has been shown to be very effective for collapsing cross-spectral data over a range of Reynolds numbers in both flat-plate and pipe flows. Farabee and Casarella (1991) determined the parameter ranges over which the similarity scaling was valid. Corcos' cross-spectral model can be transformed analytically in closed form to obtain a model for the wavenumber–frequency spectrum. However, this involves the approximation that the convection velocity is constant. Keith and Abraham (1997) showed that when the variation in convection velocity with frequency and spatial sampling is taken into account the spectral levels at high and low wavenumbers (with respect to the convective ridge) are significantly changed. The simple, closed form analytical model obtained is therefore inadequate. Another approach taken by various investigators has been to add parameters to Corcos' cross-spectral model in an attempt to correct these levels. However, that approach is purely empirical, deviates from the basic model Corcos proposed, and is not based on turbulence physics. Therefore, there is no closed form expression related to Corcos' cross-spectral model which is based on boundary layer physics and accurately represents the wavenumber spectra at all wavenumbers.

Chase (1987) developed a semi-empirical model of the wavenumber–frequency spectra, $\Phi(k_x,k_z,\omega)$, based upon the measurements of Bull (1967) as well as velocity measurements in the boundary layer. This model is expressed as

$$\Phi(k_x,k_z,\omega) = \frac{\rho^2 u_\tau^3}{[K_+^2 + (b\delta)^{-2}]^{5/2}} \left\{ C_T K^2 \left[\frac{K_+^2 + (b\delta)^{-2}}{K^2 + (b\delta)^{-2}} \right] + C_M k_x^2 \right\}, \tag{2}$$