Ocean Engineering 36 (2009) 1119–1127

Contents lists available at ScienceDirect

Ocean Engineering

journal homepage: www.elsevier.com/locate/oceaneng

Sinkage and trim of two ships passing each other on parallel courses

Tim Gourlay *

Centre for Marine Science and Technology, Curtin University, Western Australia GPO Box U1987, Perth, WA 6845, Australia

ARTICLE INFO

Article history:
Received 21 November 2008
Accepted 9 June 2009
Available online 21 June 2009

Keywords:
Sinkage
Trim
Squat
Passing
Overtaking
Shallow
Slender-body

ABSTRACT

A theoretical method is used to predict the sinkage and trim of two moving ships as they pass each other, either from opposite directions, or as one ship overtaking the other. The description is simplified to open water of shallow constant depth. The method is based on linear superposition of slender-body shallow-water flow solutions. It is shown that even for head-on encounters, oscillatory heave and pitch effects are small, and sinkage and trim can be calculated using hydrostatic balancing. Results are compared to available experimental results, and applied to an example situation of a containership and bulk carrier in a head-on or overtaking encounter. Using dimensional analysis, simple approximate formulae are then developed for estimating the maximum sinkage of two similar vessels in a passing encounter.

1. Introduction

Passing manoeuvres of ships in shallow water can produce significant sway and yaw motions of each vessel, which can be dangerous if not properly understood and allowed for. Much research has been done into calculating these sway and yaw motions. Tuck and Newman (1974) developed a slender-ship method for calculating sway forces and yaw moments for two ships moving on parallel courses in deep water. The ships could each have arbitrary speeds, so the solution was valid for head-on encounters, overtaking manoeuvres, or for one ship stationary. King (1977) included the effect of horizontal circulation, applying a Kutta condition at each ship's stern. Yeung (1978) developed a shallow-water method, including the effect of circulation, to calculate sway forces and yaw moments on each ship. Davis and Geer (1982) developed an alternative method for calculating the slender-body sway forces and yaw moments, based on asymptotic analysis. Further numerical work to predict sway forces and yaw moments was done by Kijima (1987), while Brix (1993) developed expressions for the maximum sway forces and yaw moments during an overtaking manoeuvre. Calculated sway forces and yaw moments were used to define the limits of control during passing or overtaking manoeuvres in Xu et al. (2008).

The specific problem of *vertical* motions of a moving ship, due to another passing ship, has received comparatively little attention. Yeung (1978) found analytically that the dominant heave force and pitch moment were due to linear superposition of the pressure fields produced by each ship. The circulation around each vessel, while important for sway forces and yaw moments, was found to have only a secondary effect on heave and pitch.

An experimental investigation into the transient sinkage and trim of passing model ships was undertaken by Dand (1981), involving the use of two independent towing carriages. These experiments showed the large changes in sinkage and trim that can occur for close-passing manoeuvres, and the resulting increase in grounding risk.

In this article, we start with the theoretical basis developed by Yeung (1978) for two ships moving on parallel courses, and calculate sinkage and trim for example ships through passing manoeuvres. Dimensional analysis will then be used to develop simple formulae for estimating the maximum sinkage of similar vessels during a passing manoeuvre.

2. Theoretical method

The slender-body shallow-water method is based on the theory of Tuck (1966) for a single ship. For simplicity we shall here consider the case of open water with constant depth, and assume the ships are passing on parallel courses from opposing directions. The ships are labelled "Base Ship" and "Passing Ship". The geometry is shown in Fig. 1, in the earth-fixed coordinates (x,y).

The y-coordinate is chosen such that the centreline of the base ship lies on $y=0$, while the centreline of the passing ship lies on $y=y_P$. We assume that y_P is large compared to each ship's beam, and of similar order to each ship's length. In this way, each ship

* Tel.: +61 8 9266 7380; fax: +61 8 9266 4799.
E-mail address: T.Gourlay@cmst.curtin.edu.au

0029-8018/$ - see front matter © 2009 Elsevier Ltd. All rights reserved.
doi:10.1016/j.oceaneng.2009.06.003

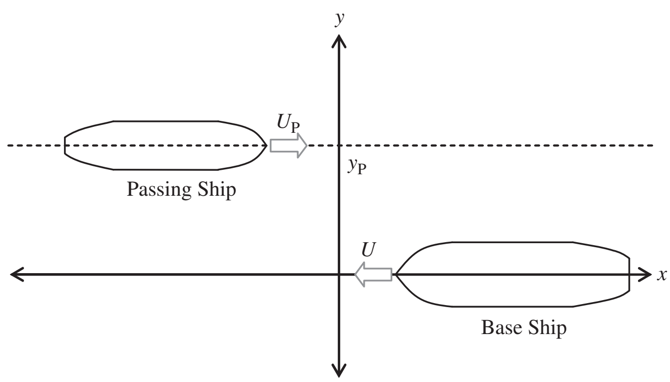

Fig. 1. Coordinate system and notation.

can be considered to lie entirely in the far field of the other vessel, as described in Yeung (1978).

The ship speeds U and U_P are assumed constant. As noted in the Dand (1981) experiments, head-on encounters and particularly overtaking encounters produce changes in resistance which translate into changes in ship speed at constant engine RPM. In this article however, this effect will not be included.

The "submerged length" of the base ship is L, which is the distance from the foremost part of the submerged hull (e.g., the front of the bulb, if present) to the aftmost part of the submerged hull. The submerged length is sometimes termed the "Length Overall Submerged" (L_{os}), but the subscripts will be omitted here, and the submerged length of the passing ship denoted L_P.

For mathematical convenience, "submerged midships" is midway between the foremost and aftmost points of the submerged hull on each ship. The (x,t) coordinates are chosen such that the submerged midships of both ships pass through $x=0$ at time $t=0$.

Following Tuck (1966), the hydrodynamic pressure field (pressure above hydrostatic) around the base ship can be written as

$$p = -\frac{\rho U^2}{2\pi h\sqrt{1-F_h^2}}\int_{-L/2}^{L/2}\frac{S'(\xi)(X-\xi)}{(X-\xi)^2+(1-F_h^2)y^2}\,d\xi. \tag{1}$$

Here h is the undisturbed water depth, assumed constant, and the depth-based Froude number is

$$F_h = \frac{U}{\sqrt{gh}}. \tag{2}$$

X is a ship-fixed coordinate centred on the ship's submerged midships, with $X=x+Ut$ if the midships pass through $x=0$ at time $t=0$. $S(\xi)$ is the hull cross-sectional area at station ξ, with the forward extremity of the submerged hull at $\xi=-L/2$ and the aft extremity at $\xi=L/2$. The primed $S'(\xi)$ denotes the derivative $dS/d\xi$. The section area is calculated at the static floating position, since to leading order the pressure field is unaffected by the dynamic sinkage and trim of the ship.

Linear superposition of the pressure fields due to each ship, as proposed by Yeung (1978), yields the following expression for the pressure on the base ship:

$$p = -\frac{\rho U^2}{2\pi h\sqrt{1-F_h^2}}\int_{-L/2}^{L/2}\frac{S'(\xi)}{X-\xi}\,d\xi - \frac{\rho U_P^2}{2\pi h\sqrt{1-F_P^2}}\int_{-L_P/2}^{L_P/2}\frac{-S'_P(\zeta)(X+X_{CC}+\zeta)}{(X+X_{CC}+\zeta)^2+(1-F_P^2)y_P^2}\,d\zeta. \tag{3}$$

The passing ship integral in the second term has $S_P(\zeta)$ defined from the bow to the stern, as for the base ship, with the forward extremity at $\zeta=-L_P/2$ and aft extremity at $\zeta=L_P/2$. The depth-based Froude number of the passing ship is

$$F_P = \frac{U_P}{\sqrt{gh}}. \tag{4}$$

The pressure field is time-dependent, through the changing longitudinal distance between ship centres (positive when approaching)

$$X_{CC} = -(U+U_P)t. \tag{5}$$

The upwards vertical force Z on the base ship is found as in Tuck (1966) to be

$$Z = \int_{-L/2}^{L/2} p(X,t)B(X)\,dX \tag{6}$$

while the bow-down trim moment about the LCF is

$$M_{LCF} = \int_{-L/2}^{L/2} p(X,t)(X-X_{LCF})B(X)\,dX. \tag{7}$$

3. Sinkage and trim

Eqs. (3), (6) and (7) give the time-dependent vertical force and trim moment on the base ship. We now seek to determine sinkage and trim. The LCF sinkage s_{LCF} is defined as the sinkage of the LCF beneath its static floating position (i.e., positive downward), in metres when using SI units. The trim θ is defined as the *change* in trim (positive bow-down) as compared to the static floating position. This is calculated in radians according to the formulae, but will be plotted in degrees for clarity.

In order to determine sinkage and trim, we must first assess whether the flow changes are sufficiently rapid to cause oscillatory heave and pitch motions of the ship. A simple quasi-steady method to determine sinkage and trim assumes that forces remain more or less in equilibrium, in which case hydrostatic balancing can be used. In that case, the upwards vertical force Z is related to the quasi-steady sinkage s_{LCF} and waterplane area A through

$$Z = -\rho g A s_{LCF}. \tag{8}$$

Similarly, M_{LCF} is related to the quasi-steady bow-down trim θ (in radians) through

$$M_{LCF} = \rho g I_{LCF}\theta, \tag{9}$$

$$I_{LCF} = \int_{-L/2}^{L/2}(X-X_{LCF})^2 B(X)\,dX. \tag{10}$$

However, if oscillatory heave and pitch are important, we need to use a seakeeping-type dynamic method to determine these. In calm water, if the LCF is close to the LCB, heave and pitch can be considered uncoupled (Bhattacharyya 1978). The sinkage equation of motion then follows the standard seakeeping form

$$(m+a_z)\frac{d^2 s_{LCF}}{dt^2} + b_z\frac{ds_{LCF}}{dt} + c_z s_{LCF} = -Z(t). \tag{11}$$

The "exciting force", which in seakeeping problems would involve wave elevation terms, is the time-dependent downward vertical force $[-Z(t)]$. The coefficients are as follows:

m	ship mass
a_z	heave added mass
b_z	heave damping coefficient
c_z	heave restoring coefficient

According to strip theory, the coefficients are calculated by summing the contributions from each hull section. For example,