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An analytic investigation of oscillations within a harbor of constant slope

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ABSTRACT

Longitudinal and transverse oscillations within a harbor of constant slope are analyzed. Based on the linear shallow water approximation, longitudinal oscillations are described with Bessel equations. Ignoring friction, oscillations are forced using the period of the incident perpendicular wave field by the method of matched asymptotics. The analytic results show that the varying depth shifts the resonant wave numbers to lower values than those for the same geometric harbor with constant depth. Furthermore, we extend the shallow water equations to a linear, weakly dispersive, Boussinesq-type equation by modifying the offshore velocity component, and then use it to investigate possible existing transverse oscillations in the harbor of constant slope. These oscillations are types of standing edge waves. Their character is quite sensitive to the boundary condition at the backwall of the harbor.

1. Introduction

Most bays and harbors in the world are open to the sea with variable depths both inside and outside. If wave motions inside a harbor are forced at one or more of its natural frequencies, the amplitudes of the harbor oscillations become large and could create unacceptable vessel motions and excessive mooring forces, leading to breaking of mooring lines for example. Evidently, the variation of water depth exerts some influence on the resonance of a harbor. However, a detailed theoretical examination of the various effects does not seem to have been conducted; thus it is the focus of this manuscript.

Harbor resonance could be affected significantly due to variable depth from three related processes. Firstly, the wave amplitudes inside the harbor will be modified through shoaling as per the bathymetry. Secondly, as the wavelength varies with water depth, the positions of nodes and antinodes are shifted, which further affects the spatial structure of wave heights in the harbor. Thirdly, due to refraction of the waves, distinct oscillation modes will occur, such as transverse oscillations with a type of standing edge wave discussed in the paper.

The study of harbor resonance is a classical subject of coastal hydrodynamics, and a number of research papers and reports have been devoted to addressing various aspects of this problem. Early work on harbor resonance was focused on the oscillations inside a simple harbor in the framework of inviscid linear long-wave theory. Miles and Munk (1961) considered harbors of arbitrary shape with narrow openings and formulated an integral equation describing the motions within the harbor by matching conditions at the entrance. Ippen and Goda (1963) applied Fourier transformation methods and obtained the solution of a rectangular harbor by matching the wave amplitude and velocity approximately at the entrance. Mei and Ünlüata (1976) obtained an analytic solution for a harbor with two rectangular basins coupled by a narrow pass by employing the method of matched asymptotic expansions. Yu (1996) examined parametrically the dissipative effects of a river channel on bay oscillations for its resonant states. These studies of the wave-induced oscillations in harbors with simple geometries have helped develop an understanding of some of the characteristics of the harbor resonance problem.

However, the shapes of actual harbors are more complex, and various numerical methods have been developed for calculating harbor oscillations. Based on the notion that the free surface oscillations are periodic and that their amplitudes vary with space but not in time, several research groups have developed numerical models using the Helmoltz equation (constant depth) and the mild-slope equation (slowly varying depth) (Chen and Houston, 1998; Lee, 1969). Due to their simplicity, these models have been used for assessing design or modification of existing harbors. Certainly, more accurate and more complicated models that can simulate the nonlinear processes of long wave generation and the subsequent response within the harbor have also been developed. These models are based primarily on Boussinesq-type equations. The effects of bottom friction and wave breaking are usually included in these models. Some of these include the FUNWAVE model where

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the governing equations are solved by the finite difference method in curvilinear grids (Kirby et al., 2003) and those solved by finite-element techniques (Losada et al., 2008; Woo and Liu, 2004).

Additionally much attention has been focused on the generation mechanisms of harbor resonance. One application of external forcing of a harbor is that of incident tsunamis which have typical periods of a few minutes to an hour, and originate from distant or local earthquakes (Kulikov et al., 1996). Tsunamis can travel across entire ocean basins, and even a small amplitude tsunami in the open sea can cause surprisingly large damage when resonance is induced in a partially enclosed harbor (Candella et al., 2008). Some researchers have suggested and further verified in their experiments that nonlinearity may transfer energy from groups of short waves to long surf beats (infragravity waves), which can in-turn excite harbor oscillations (De Girolamo, 1996; Wu and Liu, 1990). Additionally the mechanism of harbor excitation by a shear flow has been investigated (Fabrikant, 1995). In yet another triggering mechanism De Jong and Battjes (2004) identified that seiches in the Port of Rotterdam were induced by fluctuations in wind speed and atmospheric pressure.

Most of these theoretical studies on harbor resonance assumed that the water depth was constant both inside and outside the harbor. Mattioli (1978) devised an integral method of the Helmholtz equation describing the wave oscillations in a basin of variable depth. Raichlen and Naheer (1976) developed a finite difference model based on the shallow water equations to treat the wave-induced oscillations of a harbor with a variable depth and geometrical shape. To accurately investigate the influence of sloping boundaries on the response of harbors to transient long wave excitation, Zelt (1986) and Zelt and Raichlen (1990) developed finite element formulations based on a set of Boussinesq equations in the Lagrangian description. However, these solutions are implicit and need to be solved numerically. Obviously, the effect of water depth on harbor resonance is significant. Shelf resonance and edge-wave effects may play a significant and interesting role in harbor resonance (Chen et al., 2004; Olsen and Li-San, 1971). Additionally Lee et al. (2009) showed that the proper placement of a channel could induce an overall reduction of the wave field in a harbor region.

To obtain a simple parametric result of the influence of water depth on harbor resonance, the present study assumes that the open sea has a horizontal seafloor and that the harbor is rectangular, but with a planar bathymetry. Moreover there are assumed initially only longitudinal oscillations inside a rectangular harbor with planar slope and small width opening to the sea; its formulary description is given in Section 2. Additionally if the harbor width is the same order magnitude as the incident wavelength, transverse oscillations would be present in the harbor, thus its analytic formulation and solution is investigated and given in Section 3. Results are summarized in Section 4.

2. Longitudinal oscillations

2.1. Formulation and solution method

An idealized rectangular harbor with a constant slope $s=\tan\beta$ is located from $x=d$ to $x=d+L$, as shown in Fig. 1. The shore runs in the y direction; $x=0$ is where the extended virtual bottom and mean sea level intersect, and x increases offshore. The axis z is positive upward from the still water level. The floor of the open sea is horizontal, and the width of the harbor is $2b$ (from $y=-b$ to $y=b$), which is assumed first to be much smaller than the wavelength of the incident waves so that the problem becomes one dimensional (1D). Analytical solutions of water waves are generally achievable for simple bathymetry based on the long-wave equation (Huan-Wen and Yan-Bao, 2007). However, for typical conditions some analytic solutions based on the mild-slope wave equation were also derived (Suh et al., 2005; Yu and Zhang, 2003). In most harbors, the water is relatively shallow compared with the oscillation wavelength. In this case, wave motion is essentially horizontal and the vertical variation is weak. Thus we use the shallow water equations to treat this problem.

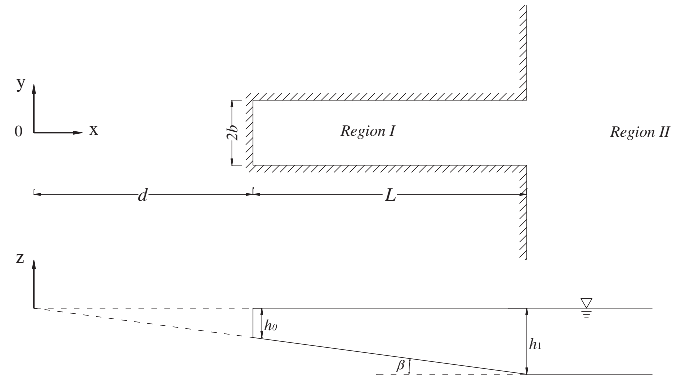

Fig. 1. Definition sketch of the harbor and coordinate systems: (a) plan view and (b) elevation view.

The linear shallow water equation for mass conservation is

$$\partial\eta/\partial t+\nabla\cdot(h\mathbf{u})=0 \tag{2.1}$$

where η is the surface elevation, t represents time, $\nabla=(\partial/\partial x,\partial/\partial y)$ is the horizontal gradient operator, $\mathbf{u}=(u,\ v)$ is the water velocity vector, and $h\ (x,\ y)$ is the still water depth. The associated momentum equation is

$$\partial\mathbf{u}/\partial t=-g\nabla\eta \tag{2.2}$$

where g is the gravitational acceleration.

In this first formulation, it is assumed that the harbor is a narrow bay; thus only longitudinal oscillations are considered while the transverse fluctuations can be ignored. Therefore the water surface displacement and the onshore–offshore velocity component inside the harbor can be written

$$\eta_I^L(x,t)=\zeta_I^L(x)\exp(\mathrm{i}\omega t)$$
$$u_I^L(x,t)=U_I^L(x)\exp(\mathrm{i}\omega t) \tag{2.3}$$

where ω is the angular frequency of the incident wave, the subscript I denotes the inner/harbor region, the superscript L represents longitudinal effects, and $i=\sqrt{-1}$. As the water depth can be written as $h=sx$, (2.1) and (2.2) can be combined to yield

$$\frac{x}{s}\frac{\mathrm{d}^2\zeta_I^L}{\mathrm{d}x^2}+\frac{1}{s}\frac{\mathrm{d}\zeta_I^L}{\mathrm{d}x}+\frac{\omega^2}{gs^2}\zeta_I^L=0 \tag{2.4}$$

Using the transformation

$$\tau=2\omega\sqrt{x/gs} \tag{2.5}$$

(2.4) may be rewritten in canonical form as

$$\mathrm{d}^2\zeta_I^L/\mathrm{d}\tau^2+1/\tau\cdot\mathrm{d}\zeta_I^L/\mathrm{d}\tau+\zeta_I^L=0 \tag{2.6}$$

which is a zero order Bessel equation. The corresponding solutions are

$$\zeta_I^L=C[J_0(\tau)+RN_0(\tau)] \tag{2.7}$$

where J_0 and N_0 are zero order Bessel functions of the first and second kinds, respectively, and C and R are constants, which can be obtained from the boundary condition and matching relations at the harbor–sea interface.

Implementing the no-flux condition at the backwall $x=d$, implies that the surface slope is zero there, or

$$\mathrm{d}\zeta_I^L/\mathrm{d}x=0|_{x=d} \tag{2.8}$$