Ocean Engineering 38 (2011) 748–762

Contents lists available at ScienceDirect

Ocean Engineering

journal homepage: www.elsevier.com/locate/oceaneng

Estimation of extreme slamming pressures using the non-uniform Fourier phase distributions of a design loads generator

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ARTICLE INFO

Article history:
Received 9 November 2009
Accepted 7 December 2010
Editor-in-Cheif: A.I. Incecik
Available online 4 March 2011

Keywords:
Non-uniform phase distribution
Modified gaussian phase distribution
Extreme values
Slamming pressure

ABSTRACT

Slamming pressures are predicted using a nonlinear ship motion program whose input is an ensemble of short wave trains tailored to produce a large, linear pitch response. These short wave trains are calculated via a design methodology that first creates short time series containing a specified, large ship response and then back-calculates the incident wave trains using linear systems theory. The background simulations and theory used to create these short time series are presented here. Monte Carlo simulation of moderately rare events of a random process indicate the random Fourier component phase PDFs are non-uniform, non-identically distributed, and dependent on the rarity of the target event. These PDFs are modeled using a single parameter, Modified Gaussian distribution and used to generate design time series with a given expected value at a specific time. To predict rare events without resorting to Monte Carlo simulation, the parameters of the Modified Gaussian distributions are calculated via characteristic function comparison. The characteristic functions compare a target PDF calculated from extreme value theory to a PDF based on a discrete Fourier representation of the stochastic process with non-uniform component phases. The comparison to extreme value theory helps to quantify the risk associated with rare events.

1. Introduction

In traditional ship design, the pressure on the hull due to slamming has been included in the ship's environmental loading via a rule-based factor. For example, classification societies commonly provide guidelines of the design pressure that includes the bottom slamming as functions of length, beam, displacement, and so on. These rules are empirically based and may inadequately cover new or unconventional hull designs due to limited existing data on such hulls. Ideally, the rule-based initial estimates on slamming pressure should be further refined by computer simulations or model tests, but in practice it is difficult to simulate the conditions that lead to extreme slamming pressures.

Traditional Monte Carlo computer simulation assumes that extreme responses will eventually be recorded if a sufficiently long exposure time is simulated, either by computer or in a wave tank. However, high fidelity long exposure times are generally not feasible to simulate due to the computationally intensive, nonlinear nature of ship-wave interactions. Similarly, physical model tests are expensive and constrained by the dimensions of the wave tank resulting in necessarily short exposure times. These limitations are especially felt during the initial design process, when many different designs must be swiftly evaluated to find the optimal hull design. Therefore, research into generating large waves and/or responses in an efficient manner has been performed in order to provide ensembles with short exposure times suitable for use in computer simulation or model tests.

Previously, generation of short time series with large waves and/or responses has primarily focused on the most likely linear representation of a wave (Tromans et al., 1991; Steinhagen, 2002; Clauss et al., 2004) or response (Adegeest et al., 1998; Pastoor, 2002; Clauss and Hennig, 2003). Also, Jensen and Pedersen (2006) investigated the most likely wave episode leading to parametric roll, a nonlinear response. These methods produce a given instance of a large wave or response, but the particular behavior of the system leading to and from that instance is lost due to the averaging effect of finding the most likely representation. Also, it is difficult to generate an ensemble of sample time series containing the large wave or response as these methods focus on finding only the single most likely large wave or response.

The current work seeks, in part, to determine a method of directly estimating an ensemble of sample time series that, when averaged at each time step, produce the most likely large wave or response. Assuming stationarity and ergodicity, a random process, $x(t)$, may be approximated as

$$x(t) \cong \sum_{j=1}^{N} \sqrt{2S^{+}(\omega_j)\Delta\omega}\cos(\omega_j t + \varepsilon_j) \quad (1)$$

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doi:10.1016/j.oceaneng.2010.12.008

where $S^+(\omega)$ is the single-sided spectrum that describes the process, N is the number of harmonic components, ω_j is the jth frequency, and ε_j is the random phase angle associated with ω_j and is uniformly distributed between $-\pi$ and π. Given a maximum value, α, at time $t=0$ ($x(0)=\alpha$), Lindgren (1970) analytically derived the expected (i.e. average) time series, $\bar{x}(t)$, as $\alpha\rho(t)$ where $\rho(t)$ is the unit autocorrelation function,

$$\rho(t)=\Re\left\{\frac{1}{\sigma^2}\int_0^\infty S^+(\omega)e^{i\omega t}\,d\omega\right\} \quad \text{continuous spectrum}$$
$$\approx\frac{1}{\sigma^2}\sum_{j=1}^{N}S^+(\omega_j)\Delta\omega\cos\omega_j t \quad \text{discrete spectrum} \tag{2}$$

and σ is the area under the spectrum,

$$\sigma^2=\int_0^\infty S^+(\omega)\,d\omega \tag{3}$$

This result was also found numerically by Boccotti (1983) and is the basis for Tromans' work cited above. However, this average time series cannot be related to the overall statistics characterizing α, nor does it capture the behavior of the process before and after the occurrence of a single realization of the maximum. More importantly, the average time series cannot be related back to a deterministic ensemble of individual instances of $x(t)$. Moreover, this representation of an extreme event cannot be used with linear theory to relate instances of extreme ship response back to the corresponding incoming wave as all information about the phases that created that particular response, ε_j, is lost.

Dietz et al. (2004) created the Conditional Random Response Wave (CRRW) approach to address the lack of time series variability. The CRRW method approximates a random process, $x(t)$, slightly differently than Eq. (1)

$$x(t)_{CRRW}\approxeq\sum_{j=1}^{N}V_j\cos(\omega_j t)+W_j\sin(\omega_j t) \tag{4}$$

where V_j and W_j are independent, standard normal random variables. The CRRW method then generates an ensemble of random time series containing a specified large ship response by conditioning V_j and W_j such that the desired response value appears as a peak (or trough) at $t=0$. The CRRW method has also been compared to towing tank experiments by Drummen and Moan (2007) for midship vertical hogging bending moment.

In practice, Eqs. (1) and (4) are equivalent: a random process is approximated by summing independent sinusoidal components. Eq. (1) uses one set of components each with a deterministic amplitude ($\sqrt{2S^+(\omega_j)\Delta\omega}$), calculated from the process' spectrum, and a uniform random phase angle (ε_j). Eq. (4) uses two sets of components each with a random amplitude, calculated from the process' spectrum (V_j and W_j). However, the conditioning process in the CRRW method theoretically affects both V_j and W_j. There is concern that the conditioning could inadvertently change the response spectrum through changing V_j and W_j.

To retain the ability to create an ensemble of $x(t)$ for use in linear ship-wave theory while preserving the integrity of the response spectrum, Troesch (1997) hypothesized that time series containing a given maximum could be generated using a single Gaussian-type distribution for the component phase angles. Later efforts to investigate this hypothesis by preserving the phase information from Fast Fourier Transforms proved inconclusive (Alford et al., 2005). The phase distributions varied widely depending on the time step, record length, and the subset of components used to generate the phase histograms. Therefore, a new set of simulations were undertaken to isolate the effects of the individual parameters. The formulation of these simulations and their results presented in this paper are the background of the design methodology presented in Alford and Troesch (2009). This methodology, here defined as the Design Loads Generator, is applied to create short wave trains that lead to large pitch, a ship response that can be supposed to lead to large slamming pressures. These short wave trains are then used as input to the nonlinear Large Amplitude Motion Program (LAMP) (Lin et al., 2008) to predict design slamming pressures.

2. Monte Carlo simulation of random processes

As previously mentioned in the application of the current theory (Alford and Troesch, 2009), a random process with an associated single-sided frequency spectrum, $S^+(\omega)$, may be approximated by the summation of a finite number of components:

$$x(t)\approxeq\sum_{j=1}^{N}a_j\cos(\omega_j t+\varepsilon_j) \tag{5}$$

where

$$a_j=\sqrt{2S^+(\omega_j)\Delta\omega} \tag{6}$$

and ε_j is a random phase angle, typically uniformly distributed between $-\pi$ and π. The random process can also be described in terms of the moments of its frequency spectrum:

$$m_k=\int_{-\infty}^{\infty}\omega^k S^+(\omega)\,d\omega \tag{7}$$

The largest value of $x(t)$ that this model can generate is

$$x_{max}=\sum_{j=1}^{N}a_j \tag{8}$$

The random process, $x(t)$, is assumed to be stationary and ergodic. Therefore, the probability density function (PDF) of $x(t)$ is also assumed to be a zero-mean, Gaussian distribution,

$$f_x(x)=\frac{1}{\sigma\sqrt{2\pi}}e^{-x^2/2\sigma^2} \tag{9}$$

where

$$\sigma^2=\frac{1}{2}\sum_{j=1}^{N}a_j^2$$

and the cumulative density function (CDF) of $x(t)$ is

$$F_x(x)=\Phi\left(\frac{x}{\sigma}\right) \tag{10}$$

where Φ is the general CDF for a Gaussian random variable. For sufficiently large N, the approximation of $x(t)$ in Eq. (5) (the right-hand side) can also be considered to be a zero-mean, Gaussian process.

3. Conditions that cause extreme events

A typical time series generated by Eq. (5) is shown in Fig. 1. Consider now an event x_1 that occurs at time t_1 (Fig. 1). x_1 is defined as

$$x_1\equiv x(t_1)=\sum_{j=1}^{N}a_j\cos(\omega_j t_1+\varepsilon_j) \tag{11}$$

x_1 is a random sample of the Gaussian process $x(t)$. Therefore, x_1 is a random variable with the same Gaussian distribution as $x(t)$. Since $x(t)$ is considered to be a stationary and ergodic process, statistics related to the distribution of x_1 can be considered to be equivalent to statistics of $x(t)$. The time series that contains x_1 at time $t=0$ may be constructed utilizing the following change of variables:

$$t'=t-t_1 \tag{12}$$