Ocean Engineering 37 (2010) 1048–1060

Contents lists available at ScienceDirect

Ocean Engineering

journal homepage: www.elsevier.com/locate/oceaneng

Bending of orthotropic super-elliptical plates on intermediate point supports

Murat Altekin *

Department of Civil Engineering, Yildiz Technical University, 34210 Esenler, Istanbul, Turkey

ARTICLE INFO

Article history:
Received 13 August 2009
Accepted 31 March 2010

Keywords:
Intermediate
Point support
Super-elliptical
Orthotropic
Plate
Optimum
Ritz method
Deflection

ABSTRACT

Bending analysis of orthotropic super-elliptical plates of uniform thickness was investigated. Optimum location of the point supports was searched by minimizing the maximum absolute deflection. The support location which minimizes the bending moments at the supports was reported. The Ritz method was used and the total potential energy functional was modified by introducing the Lagrange multipliers to improve the accuracy of the stress resultants. The deflection and the bending moments computed at various points for a large variety of plate shapes ranging from an ellipse to a rectangle were checked with those of rectangular and elliptical plates. Good agreement was obtained for both cases. The structural response was found to be sensitive to support position.

1. Introduction

Both the shape and the position of supports have practical importance in engineering applications. Supports do not only hold a structure properly, but they may be used to improve the structural performance as well (Wang, 2006). The stiffness and the load-carrying capacity of structures spanning large areas may be increased by utilizing intermediate supports (Wang et al., 1992). Although plates resting on isolated points such as telescope mirrors, solar panels, printed circuit boards, slabs or platforms supported by columns are frequently encountered in optics and structural design, in the literature relatively less attention has been paid to the studies on the bending of plates with intermediate supports (Nelson et al., 1982; Gutierrez and Laura, 1995; Chinnaboon et al., 2007; Xu and Zhou, 2009). In comparison with fully clamped or simply supported boundaries, owing to the mathematical difficulties in handling more complicated boundary conditions, the studies on the bending analysis of point-supported plates have been scanty, and generally numerical methods have been applied in these studies. The published studies have mostly been modelled by the classical theory of thin plates and have generally focused on uniformly loaded isotropic plates supported at isolated points along the edges (Lee and Ballesteros, 1960; Pan, 1961; Cheung et al., 1988; Wang and Lim, 2000; Altekin and Altay, 2008). This is probably partly because uniformly loaded plates are essential parts of numerous machines and structures and partly because the classical plate bending theory suffices if the plate is thin, elastic and homogeneous (Wang, 2004a).

Plates of anisotropic nature are commonly used in bridge decks, wings of aircrafts, ship hulls, and missiles. However, apart from a few exceptions anisotropic plate bending problems have not been extensively studied in academic areas (Akbarov and Kocaturk, 1997; Ozcelikors et al., 1997; Laura and Rossit, 1998; Rajamohan and Raamachandran, 1999; Mbakogu and Pavlovic, 2000; Vasilenko, 2002; Dong et al., 2004; Setoodeh and Karami, 2004; Chen and Nie, 2004; Gawandi et al., 2008).

Lee et al. (1971) proposed an approximate solution for corner supported orthotropic rectangular plates. As the deflection function they used the same truncated two-dimensional polynomial of degree four in the solution like Lee and Ballesteros (1960) did. Shanmugam et al. (1988) presented an approximate method for the bending behavior of rhombic orthotropic plates supported at the corners by minimizing the total potential energy. They considered the same polynomial used by Lee and Ballesteros (1960) as the deflection function. Raamachandran and Reddy (1989) employed the charge simulation method to investigate the static behavior of point-supported circular plates. Zhi-qing and Jin-xi (1992) proposed a closed series solution for the bending of point-supported rectangular thin plates and presented the numerical results of an isotropic square plate under uniform pressure and a central concentrated load. Wang et al. (1994) examined axisymmetric plates supported on multiple internal concentric rings. Their objective was to maximize the load-carrying capacity of the plate by positioning the supports. Craig and Boulet (1999) analyzed the transverse deflections of a

* Tel.: +90 2123835148; fax: +90 2123835133.
E-mail addresses: altekin@yildiz.edu.tr, murataltekin@yahoo.com.

0029-8018/$ - see front matter © 2010 Elsevier Ltd. All rights reserved.
doi:10.1016/j.oceaneng.2010.03.015

uniformly loaded thin circular plate supported by equally spaced point supports. The reciprocal theorem was applied by Yuhong (2000) to solve the bending of a point-supported rectangular plate subjected to a concentrated load. Wang (2004a) studied the optimum size of the support for a uniformly loaded isotropic circular plate analytically and numerically. The support positions were optimized to minimize the maximal deflections of beams or plate structures by Wang (2004b).

Super-elliptical plates include a large variety of plate shapes ranging from an ellipse to a rectangle with rounded corners. However, the number of studies dealing with them has been rather limited. The present literature[1] has basically focused on the approximate solutions of vibration and buckling analysis (Wang et al., 1994; Liew et al., 1998; Chen et al., 1999; Chen and Kitipornchai, 2000; Wu and Liu, 2005). The current work was motivated by the lack of contributions on the static analysis of orthotropic super-elliptical plates on intermediate supports. In the study the plate which was assumed to be rectilinearly orthotropic was considered to be supported by symmetrically distributed four point supports on the diagonals. The modelling was based on the classical theory of thin plates and the Ritz method was used. Two loading cases were considered in the analysis: (1) uniform pressure, (2) a central concentrated transverse load. The optimum support location which minimizes the maximum absolute deflection was investigated in case of uniform pressure. Since from engineering standpoint the bending moments are important in design, the support location minimizing the bending moments at the supports was highlighted for the second loading case.

2. Formulation

A rectilinearly orthotropic super-elliptical plate of uniform thickness is considered in the linear elastic range. In Cartesian co-ordinates the boundary contour of the plate can be defined by the equation

$$\left(\frac{x}{a}\right)^{2k}+\left(\frac{y}{b}\right)^{2k}=1,\quad k=1,2,\ldots\infty \tag{1}$$

where a, b and k are the semi-major axis, the semi-minor axis and the super-elliptical power, respectively (Liew et al., 1998).

The strain energy of the plate due to bending (U), the potential energy of the lateral load (V_1, V_2), and the governing equation (L) can be written as (Lekhnitskii, 1968; Shanmugam et al., 1989)

$$U=\frac{1}{2}\int_{x_1}^{x_2}\int_{y_1}^{y_2}\left[D_x\left(\frac{\partial^2 w}{\partial x^2}\right)^2+2D_y\mu_x\left(\frac{\partial^2 w}{\partial x^2}\right)\left(\frac{\partial^2 w}{\partial y^2}\right)+D_y\left(\frac{\partial^2 w}{\partial y^2}\right)^2+4D_{xy}\left(\frac{\partial^2 w}{\partial x\partial y}\right)^2\right]dx\,dy, \tag{2}$$

$$V_1=-\int_{x_1}^{x_2}\int_{y_1}^{y_2}qw\,dx\,dy,\quad V_2=-Pw_P,\quad w_P=w(x=x_P;\ y=y_P) \tag{3}$$

$$L=D_x\frac{\partial^4 w}{\partial x^4}+2H\frac{\partial^4 w}{\partial x^2\partial y^2}+D_y\frac{\partial^4 w}{\partial y^4}-\eta_1 q-\eta_2\delta(x-x_P)\delta(y-y_P)P=0 \tag{4}$$

where

$$x_1=-x_2=-a,\quad y_1=-y_2=-\frac{b}{a}\sqrt[2k]{a^{2k}-x^{2k}},\quad x_P=y_P=0. \tag{5}$$

Here; w denotes the deflection, q is the uniform pressure, P is the concentrated load acting at $(x=x_P; y=y_P)$, w_P is the central deflection, δ is the Dirac delta function, and D_x, D_y, D_{xy} are the flexural and torsional rigidities which can be expressed in terms of the effective torsional rigidity H and the Poisson's ratios μ_x, μ_y by

$$H=D_1+2D_{xy},\quad D_1=\mu_x D_y=\mu_y D_x. \tag{6}$$

The bending moments (m_x, m_y), and the lateral edge forces (ν_x, ν_y) per unit length of the plate are (Szilard, 1974)

$$m_x=-D_x\left(\frac{\partial^2 w}{\partial x^2}+\mu_y\frac{\partial^2 w}{\partial y^2}\right),\quad \nu_x=-D_x\left[\frac{\partial^3 w}{\partial x^3}+\left(\frac{4D_{xy}}{D_x}+\mu_y\right)\frac{\partial^3 w}{\partial x\partial y^2}\right], \tag{7}$$

$$m_y=-D_y\left(\mu_x\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial y^2}\right),\quad \nu_y=-D_y\left[\frac{\partial^3 w}{\partial y^3}+\left(\frac{4D_{xy}}{D_y}+\mu_x\right)\frac{\partial^3 w}{\partial x^2\partial y}\right]. \tag{8}$$

The scalar indicators η_1 and η_2 are determined depending on the loading (Table 1). The aspect ratio c, and the non-dimensional counterparts of x, y, x_2, y_2, D_y, D_{xy} are introduced by

$$X=\frac{x}{a},\ Y=\frac{y}{b},\ X_2=1,\ Y_2=\sqrt[2k]{1-X^{2k}} \tag{9}$$

$$D_y^*=\frac{D_y}{D_x},\ D_{xy}^*=\frac{D_{xy}}{D_x},\ c=\frac{a}{b}. \tag{10}$$

Hence, substituting the non-dimensional variables into Eqs. ((2)–(4))

$$U_1=\frac{1}{2}\frac{q^2a^5b}{D_x}\int_0^{X_2}\int_0^{Y_2}\left[\left(\frac{\partial^2 W}{\partial X^2}\right)^2+2D_y^*\mu_x c^2\left(\frac{\partial^2 W}{\partial X^2}\right)\left(\frac{\partial^2 W}{\partial Y^2}\right)+D_y^*c^4\left(\frac{\partial^2 W}{\partial Y^2}\right)^2+4D_{xy}^*c^2\left(\frac{\partial^2 W}{\partial X\partial Y}\right)^2\right]dX\,dY, \tag{11}$$

$$U_2=\frac{1}{2c}\frac{P^2a^2}{D_x}\int_0^{X_2}\int_0^{Y_2}\left[\left(\frac{\partial^2 W}{\partial X^2}\right)^2+2D_y^*\mu_x c^2\left(\frac{\partial^2 W}{\partial X^2}\right)\left(\frac{\partial^2 W}{\partial Y^2}\right)+D_y^*c^4\left(\frac{\partial^2 W}{\partial Y^2}\right)^2+4D_{xy}^*c^2\left(\frac{\partial^2 W}{\partial X\partial Y}\right)^2\right]dX\,dY, \tag{12}$$

$$V_1=-\frac{q^2a^5b}{D_x}\int_0^{X_2}\int_0^{Y_2}W\,dX\,dY,\quad V_2=-\frac{1}{4}\frac{P^2a^2}{D_x}W_P, \tag{13}$$

$$L_1=\left[\frac{\partial^4 W}{\partial X^4}+c^2(2\mu_x D_y^*+4D_{xy}^*)\frac{\partial^4 W}{\partial X^2\partial Y^2}+c^4D_y^*\frac{\partial^4 W}{\partial Y^4}-1\right]=0, \tag{14}$$

$$L_2=\left[\frac{\partial^4 W}{\partial X^4}+c^2(2\mu_x D_y^*+4D_{xy}^*)\frac{\partial^4 W}{\partial X^2\partial Y^2}+c^4D_y^*\frac{\partial^4 W}{\partial Y^4}-\frac{1}{4}c\,\delta(X-X_P)\delta(Y-Y_P)\right]=0 \tag{15}$$

are obtained. Here, the numerical subscripts indicate to which case number the expressions on the left hand side shown in Eqs. (11)–(15) correspond. Due to the symmetry of the plate geometry, a quarter of the plate is used in the computations. Therefore, both V_2 and the last term of L_2 are divided by four as shown in Eqs. (13), (15)). Since the point load acts at the center of the plate, X_P and Y_P are equal to zero.

[1] The other related articles can be seen in Altekin (2010) and the references cited therein.