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Scattering of obliquely incident water waves by partially reflecting non-transmitting breakwaters

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ABSTRACT

In the present paper, analytic solutions are derived for scattering of water waves obliquely incident to a partially reflecting semi-infinite breakwater or breakwater gap. In order to examine the correctness of the derived solutions, they are compared with the solutions derived by McIver (1999) and Bowen and McIver (2002) for a semi-infinite breakwater and a breakwater gap, respectively, in the case of perfect reflection. The derived analytic solutions are used to investigate the effect of reflection coefficient of the breakwater and wave incident angle upon the tranquility at harbor entrance. The tranquility is deteriorated by the reflected waves as the reflection coefficient increases and as waves are incident more obliquely.

1. Introduction

Water wave scattering by semi-infinitely long breakwaters has long been a subject of coastal and ocean engineering researchers. Penney and Price (1952) proposed an analytic solution for diffracted waves around a semi-infinitely long impermeable breakwater based on the Sommerfeld's (1896) solution for diffraction of light. They also obtained the solution for the waves transmitted through a gap in a breakwater by superposing the solutions for the semi-infinite breakwaters. Their results have been cited and presented as a table or diffraction diagrams in many textbooks and manuals, e.g., Wiegel (1964) and Coastal Engineering Research Center (1984). Recently, Yu (1995) derived the boundary condition for a thin porous wall based on the formulation of Sollitt and Cross (1972), and used it to find an approximate solution for diffraction of water waves normally incident to a semi-infinite permeable breakwater. More recently, McIver (1999) extended the Yu's solution to obliquely incident waves using the Wiener–Hopf technique. On the other hand, Bowen and McIver (2002) derived an analytic solution for diffraction by a gap in a permeable breakwater. They formulated the problem in terms of an integral equation obtained by an application of Green's theorem that involves a new Green's function for permeable barriers. The integral equation is solved numerically, and the solution is used to obtain the diffraction coefficient that describes the far-field behavior of the scattered waves.

The solution of Penney and Price (1952) could be used for a vertical caisson breakwater, whilst those of Yu (1995), McIver (1999), and Bowen and McIver (2002) could be used for a rubble mound breakwater or any other permeable breakwaters such as curtain wall or pile breakwaters. Nowadays, to reduce wave reflection from, and impulsive wave pressure acting on, a vertical caisson breakwater, a horizontally composite breakwater (i.e. a vertical caisson breakwater covered with wave-energy-dissipating concrete blocks, and named by Takahashi 1997) or a perforated-wall caisson breakwater is often used, which has a porous front and a solid back. Such type of breakwaters can also improve the conditions for vessel navigation in harbor entrance area, resulting in a safer approach to a harbor entrance or maneuvering within the entrance itself (see McBride et al., 1994). Very recently, following the approach of Penney and Price (1952), Suh and Kim (2008) derived analytic solutions for water wave scattering by a semi-infinite breakwater or a breakwater gap of partial reflection. In the present study, their solution is extended to obliquely incident waves. The derived solutions are then used to investigate the effect of reflection coefficient of the breakwater and wave incident angle upon the tranquility at harbor entrance.

Though not directly related to the present study, there are several studies for calculating the reflection coefficient of obliquely incident waves by a perforated-wall caisson breakwater using eigenfunction expansion methods (e.g., Suh and Park 1995; Li et al. 2002; Teng et al. 2004). The reflection coefficient calculated by the

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eigenfunction expansion methods could be used as one of the input data which are needed in the present solution.

2. Analytic solutions

2.1. Semi-infinite breakwater

As shown in Fig. 1, a semi-infinitely long breakwater of partial reflection on both sides stands in water of uniform depth h. Cartesian coordinates x, y, and z are chosen with origin by the mean free surface at the tip of the breakwater, the x- and y-axes lie in a horizontal plane and the z-axis is directed vertically upward. The breakwater is placed along the positive x-axis. A regular wave train is incident to the breakwater at an angle θ_0 counterclockwise from the positive x-axis.

Assuming incompressible fluid and irrotational flow motion, the velocity potential exists, which satisfies the Laplace equation. Linearizing the free-surface boundary conditions, the following boundary value problem for the velocity potential $\Phi(x,y,z,t)$ is obtained:

$$\frac{\partial^2\Phi}{\partial x^2}+\frac{\partial^2\Phi}{\partial y^2}+\frac{\partial^2\Phi}{\partial z^2}=0 \tag{1}$$

$$\frac{\partial\Phi}{\partial z}=0 \quad \text{at } z=-h \tag{2}$$

$$-\frac{\partial\Phi}{\partial z}=\frac{\partial\eta}{\partial t} \quad \text{at } z=0 \tag{3}$$

$$-\frac{\partial\Phi}{\partial t}+g\eta=0 \quad \text{at } z=0 \tag{4}$$

where $\eta(x,y,t)$ is the free surface elevation and g is the gravitational acceleration. In Fig. 1, $A_1(0<\theta<\theta_0)$ is the protected area, $A_2(\theta_0<\theta<2\pi-\theta_0)$ is the area which is not directly influenced by the breakwater, and $A_3(2\pi-\theta_0<\theta<2\pi)$ is the reflection area.

First, the analytic solution is obtained for the case of normal incidence ($\theta_0=\pi/2$). The solution for oblique incidence can be obtained by the coordinate transformation of the solution for normal incidence, as will be explained later. The velocity potential satisfying the periodicity in time and the no-flow bottom boundary condition is represented by

$$\Phi(x,y,z,t)=A\cosh[k(z+h)]F(x,y)e^{i\omega t} \tag{5}$$

where k and ω are the wave number and wave angular frequency, respectively, and $F(x,y)$ is a complex function. Substituting Eq. (5) into Eqs. (3) and (4) gives the dispersion relationship

$$\omega^2=gk\tanh(kh) \tag{6}$$

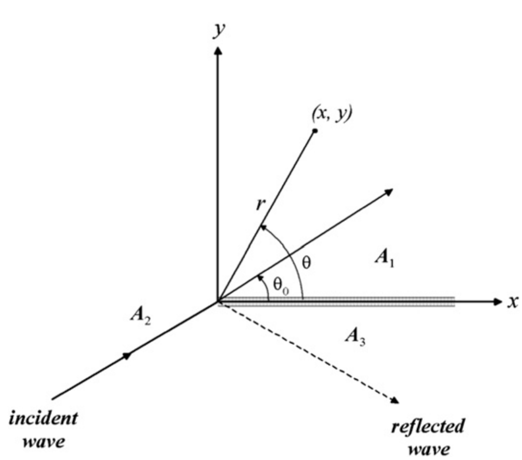

Fig. 1. Definition sketch of wave motion around a semi-infinite breakwater.

On the other hand, substituting Eq. (5) into the Laplace equation yields the Helmholtz equation in $F(x,y)$:

$$\frac{\partial^2F}{\partial x^2}+\frac{\partial^2F}{\partial y^2}+k^2F=0 \tag{7}$$

To solve this equation, we closely follow the approach of Sommerfeld (1896), which is also summarized in Lamb (1945, p. 538). The general solution to the preceding equation can be expressed as the sum of two solutions:

$$F(x,y)=e^{-iky}F_1(x,y)+e^{iky}F_2(x,y) \tag{8}$$

Since the procedure for solving the equation is the same for both solutions, the procedure is described only for one solution. Substituting the first solution into Eq. (7) gives

$$\frac{\partial^2F_1}{\partial x^2}+\frac{\partial^2F_1}{\partial y^2}-2ik\frac{\partial F_1}{\partial y}=0 \tag{9}$$

It is convenient to introduce the following parameters:

$$kx=\xi^2-\psi^2, \quad ky=2\xi\psi \tag{10}$$

$$kr=\xi^2+\psi^2 \tag{11}$$

where $r=\sqrt{x^2+y^2}$ is the distance from the origin of the coordinate system. We easily find

$$\frac{\partial\xi}{\partial y}=\frac{\psi}{2r}, \quad \frac{\partial\psi}{\partial y}=\frac{\xi}{2r} \tag{12}$$

$$\frac{\partial\xi}{\partial x}=\frac{\xi}{2r}, \quad \frac{\partial\psi}{\partial x}=-\frac{\psi}{2r} \tag{13}$$

$$\frac{\partial^2\xi}{\partial x^2}+\frac{\partial^2\xi}{\partial y^2}=0, \quad \frac{\partial^2\psi}{\partial x^2}+\frac{\partial^2\psi}{\partial y^2}=0 \tag{14}$$

Using these relations, Eq. (9) can be expressed as an equation of ξ and ψ:

$$\frac{\partial^2F_1}{\partial\xi^2}+\frac{\partial^2F_1}{\partial\psi^2}-4i\left(\psi\frac{\partial F_1}{\partial\xi}+\xi\frac{\partial F_1}{\partial\psi}\right)=0 \tag{15}$$

This equation can be transformed into an ordinary differential equation of a single variable ρ by using the relation $u=f(\xi-\psi)=f(\rho)$:

$$\frac{d^2f}{d\rho^2}+2i\rho\frac{df}{d\rho}=0 \tag{16}$$

Solving this equation, the following solution can be obtained:

$$F_1=\alpha+\beta\int_0^{\xi-\psi}e^{-i\rho^2}\,d\rho \tag{17}$$

Similarly, the second solution in Eq. (8) can be obtained as

$$F_2=\gamma+\delta\int_0^{\xi+\psi}e^{-i\rho^2}\,d\rho \tag{18}$$

The unknowns α, β, γ, and δ can be calculated by applying the partial reflection condition at the breakwater:

$$\frac{\partial F}{\partial\mathbf{n}}+bF=0 \tag{19}$$

where $\mathbf{n}$ is the unit normal vector directing from water to the breakwater, and $b=b_1+ib_2$ is the complex reflection coefficient. Assuming that there is no phase difference between incident and reflected waves, we have

$$b_1=0, \quad b_2=k\sin\theta_i\frac{1-C_r}{1+C_r} \tag{20}$$

where C_r is the reflection coefficient at the breakwater, and θ_i is the angle between the incident wave direction and the breakwater crest line. In the case of normal incidence, $\mathbf{n}=y$ and $\theta_i=\pi/2$ at the front face of the breakwater, whilst $\mathbf{n}=-y$ and $\theta_i=0$ at the back