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Solution of the propagation of the waves in open channels by the transfer matrix method

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ABSTRACT

Many problems in mechanics can be solved by the use of the transfer matrix method. The use of this method in hydraulics engineering is not widespread and only limited studies are available. In this study, linearized St. Venant equations were used and the use of transfer matrix in ocean engineering was investigated for long waves in open channels, and numerical application was carried out. The results obtained through the transfer matrix method, which is quite easy to use, program and comprehend, showed similar results obtained from the characteristics method and finite differences method.

1. Introduction

St. Venant equations can be used in solving equations for wave movements in shallow waters. Since St. Venant equations are non-linear and second-degree hyperbolic, they cannot be integrated directly. By linearizing St. Venant equations can be integrated. In this study, non-linear term is usually neglected and the linearized version of St. Venant equations was used and the transfer matrix method (TMM) was considered for the purpose of obtaining a solution for the long waves in open channels.

In the TMM, which is known as the method of initial values where the aim is to convert the problem of boundary values into the problem of initial value to prevent new constant values and to express equations of the problem by initial constant values (Inan, 1968). With the help of this method, many problems in mathematics can be solved (Dimarogonas, 1996). The method can be used essentially for the solution of 1D linear differential equations; however, after a proper linearization process it can also be used to solve nonlinear problems.

The application of transfer matrix for the solution of hydraulic problems is very limited. According to the finite elements method, matrix dimensions are small, constant and independent of the number of elements. Computer programing of the method is easy and practical (Daneshfaraz and Kaya, 2007). The TMM can be used in determining movements of waves in shallow waters.

When the studies on long wave in literature are viewed, it can be seen that a series of studies were carried out. However no study on the solution of long waves using transfer matrix is available. Studies using other methods and approaches are given below:

Tsai (2002) conducted theoretical investigations on the propagation of long waves of one-dimensional, unstable, viscous and turbulent open canal currents, and discussed the effect of the Froude number on the formation of channel flows in shallow waters depending on the location and time. Shi et al. (2005) investigated the fundamental behavior of long water waves propagating through branching channels of uniform depth and width. They carried out numerical simulations based on the Boussinesq long wave model to verify the effects of width of channel branches on wave transmission and reflection. Koutitas (1983) solved the linear long wave equation by using the finite elements method. In the study, it was accepted that the flow generated a sinusoidal vibration.

Onzikua and Odai (1998) proposed the Burgers equation model for unsteady flow in open channels. In this model to simulate slow transients in wide rectangular open channels of finite length, the St. Venant equations are approximated by a single Burgers equation for flow depth. Flow velocity is expressed as a function of flow depth and its gradient to satisfy the continuity of the flow. Tsai and Yen (2001) suggested a method for linear analysis of shallow water wave propagation in open channels. In this study, the Laplace transform method is adopted to obtain first-order analytical spatiotemporal expressions of upstream and down-stream channel response function.

The methods in the discussed studies are numerical methods. However, the TMM is based on analytical solution. The TMM is used in studies of Baume et al. (1998) and Litrico and Fromion

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(2006). Baume et al. (1998) expressed a need for using linear control theory while pointing out the difficulty of complex hydraulic systems to control. They obtained a reach transfer matrix by liberation of St. Venant equations near a steady flow regime. In this study, St. Venant equations were used in a hydraulic application for the first time. However, the equations were used between the two given points only. Litrico and Fromion (2006) investigated the control of oscillating modes occurring in open channels due to the reflection of propagating waves on the boundaries. They characterized the effect of a proportional boundary control on the poles of the transfer matrix by a root locus which derived to an asymptotic result for high-frequency closed-loop poles. Baume et al., (1998) and Litrico and Fromion (2006) have solved linearized equations via the Laplace method. In these studies, separation of variables method is used.

The proposed method in this study is also extendable and applicable to study of wave interaction generated by vessels moving in either parallel or opposite directions (Wu et al., 2001).

2. Equations for long linear waves in open channels

The dynamic behavior can be described by a set of equations known as the St. Venant equations (Chaudhry, 1993):

$$\frac{\partial y}{\partial t}+D\frac{\partial u}{\partial x}+u\frac{\partial y}{\partial x}-\frac{q}{B}=0 \tag{1}$$

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}-g(S_0-Sf)+g\frac{\partial y}{\partial x}=0 \tag{2}$$

where B is the top water surface width (m), D is the hydraulic depth (m), y the water depth (m), g the gravity acceleration (m^2/s), x the longitudinal abscissa in the direction of the flow, t the time, S_0 the bottom slope, $S_{\rm f}$ the energy gradient slope, u the average velocity (m/s) and q the lateral inflow or outflow per unit length.

If there is not lateral inflow or outflow $q=0$, the St. Venant equation for very wide rectangular cross-section channels. Eq. (1) can be written as

$$\frac{\partial \xi}{\partial t}+\frac{\partial (hu)}{\partial x}=0 \tag{3}$$

For $S_{\rm f}=\tau_{\rm b}/\rho gh$, Eq. (2) can be written as

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}=-g\frac{\partial \xi}{\partial x}-\frac{\tau_{\rm b}}{\rho h}+gS_0 \tag{4}$$

where ξ is the amplitude, h is the undisturbed flow depth, ρ is the mass density of water and $\tau_{\rm b}$ is shear stress at the base and is a function of the velocity, and is described with the following equation:

$$\frac{\tau_{\rm b}}{\rho}=ku \tag{5}$$

where k is equivalent friction coefficient. If Eq. (5) is substituted in Eq. (4), Eq. (6) is obtained

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}=-g\frac{\partial \xi}{\partial x}-\frac{ku}{h}+gS_0 \tag{6}$$

For shallow water wave propagation, Eqs. (3) and (6) can be written. For gradual variations in $\xi(x,t)$ (propagation of long waves) and small variations in $h(x)$ the non-linear term $u(\partial u)/(\partial x)$ is usually neglected and the linearized version of Eqs. (3) and (6) is

$$\frac{\partial u}{\partial t}=-g\frac{\partial \xi}{\partial x}-\frac{ku}{h}+gS_0 \tag{7}$$

$$\frac{\partial \xi}{\partial t}+\frac{\partial}{\partial x}(hu)=0 \tag{8}$$

If the two equations above are combined and reorganized, a second-degree, linear, and hyperbolic equation will be obtained

$$\frac{\partial^2 \xi}{\partial t^2}=g\frac{\partial}{\partial x}\left(h\frac{\partial \xi}{\partial x}\right)-\frac{k}{h}\frac{\partial \xi}{\partial t} \tag{9}$$

If channel bottom slope is invariable, derivate of 'gS_0' is zero. The flow domain is discretized into equal elements of length Δx. The water depth is assumed constant along each element (i), $h_i=$ const. (Koutitas, 1983).

3. Solution by the TMM

In various engineering problems, as the number of constants to be determined by the use of boundary condition increases, the calculation becomes more tedious and the possibility of making errors increases. Therefore, in the formulation of such problems, ways of reducing the number of constants to a minimum are sought. The method of transfer matrix makes this possible. The main principle of this theory, which is applied to problems with one variable, is to convert all the boundary value problems into problems of initials values, and thus new constants that may result from the use of intermediate condition are eliminated. Therefore, it is a method of expressing the equations in terms of the initials constants. This method thus makes no distinction between the so-called determinate and indeterminate problems of elastomechanics (Inan, 1968).

There are a number of methods for solving the differential equations, one of which is the TMM. The TMM is ideally suited to solve mechanical systems, because only successive matrix multiplication is necessary to fit the elements together. One of the used solutions of differential equation is separation of variables (Riley et al., 1998). The method of separation of variables can be used to obtain the solution of Eq. (9). Assuming that

$$\xi(x,t)=\xi(x)\xi(t) \tag{10}$$

and substituting for ξ in Eq. (9), we obtain

$$\ddot{\xi}(t)\cdot\xi(x)-gh_i\xi''(x)\xi(t)+\frac{k}{h_i}\dot{\xi}(t)\xi(x)=0 \tag{11}$$

If simplifications are made, the following equation will be obtained:

$$\frac{\ddot{\xi}(t)+(k/h_i)\dot{\xi}(t)}{\xi(t)}=gh_i\frac{\xi''(x)}{\xi(x)}=-\alpha^2 \tag{12}$$

where α is a constant and equals to $2\pi/7$. Thus, we find that $\xi(x)$ and $\xi(t)$ satisfy the ordinary differential equations

$$gh_i\frac{\xi''(x)}{\xi(x)}=-\alpha^2 \tag{13}$$

and

$$\frac{\ddot{\xi}(t)+(k/h_i)\dot{\xi}(t)}{\xi(t)}=-\alpha^2 \tag{14}$$

then Eq. (13) can be written as

$$\xi''(x)+\frac{\alpha^2}{gh_i}\xi(x)=0 \tag{15}$$

The solution of Eq. (15) may be written as follows:

$$\xi(x)=C_1\cos\left(\frac{\alpha}{\sqrt{gh_i}}x\right)+C_2\sin\left(\frac{\alpha}{\sqrt{gh_i}}x\right) \tag{16}$$

The first derivative of $\xi(x)$ can be expressed as

$$\frac{{\rm d}\xi(x)}{{\rm d}x}=-C_1\frac{\alpha}{\sqrt{gh_i}}\sin\left(\frac{\alpha}{\sqrt{gh_i}}x\right)+C_2\frac{\alpha}{\sqrt{gh_i}}\cos\left(\frac{\alpha}{\sqrt{gh_i}}x\right) \tag{17}$$