Ocean Engineering 35 (2008) 1029–1038

Contents lists available at ScienceDirect

Ocean Engineering

journal homepage: www.elsevier.com/locate/oceaneng

Quantifying uncertainty in extreme values of design parameters with resampling techniques

Ying Li *, Dave Simmonds, Dominic Reeve

University of Plymouth, Centre for Coastal Dynamics and Engineering, School of Engineering, Drake Circus, Plymouth PL4 8AA, UK

ARTICLE INFO

Article history:
Received 22 May 2007
Accepted 28 February 2008
Available online 8 March 2008

Keywords:
Extreme value
Jackknife
Bootstrap
Resampling
Return period
Confidence interval

ABSTRACT

In this paper, a methodology for the selection of statistical models for describing the extreme wave heights on the basis of resampling techniques is presented. Two such techniques are evaluated: the jackknife and the bootstrap. The methods are applied to two high-quality datasets of wave measurements in the Mediterranean and one from the East Coast of the USA. The robustness of the estimates of the extreme values of wave heights at return periods important for coastal engineering design is explored further. In particular, we demonstrate how an ensemble error norm can be used to select the most appropriate extreme probability model from a choice of cumulative distribution functions (CDFs). This error norm is based on the mean error norm of the optimised CDF for each resampled (replicate) data series. The resampling approach is also used to present confidence intervals of the CDF parameters. We provide a brief discussion of the sensitivity of these parameters and the suitability of each model in terms of uncertainty with resampling techniques. The advantages of resampling are outlined, and the superiority of the bootstrap over the jackknife in quantifying the uncertainty of extreme quantiles is demonstrated for these records.

1. Introduction

The degree of uncertainty attached to extreme quantile estimates of environmental parameters is often large or left unquantified. The main reason for this is down to the restricted duration of most datasets. In particular, coastal and flood risk engineers are regularly required to make predictions of parameters such as extreme wave heights or storm surge based upon datasets that rarely extend beyond decadal extent. Such parameters directly determine the appropriate level of coastal protection required. Furthermore, the assessment of civil engineering infrastructure and natural defences are key input parameters when using limit state equations for flood defences in coastal areas, estuaries and rivers. It follows that lack of knowledge concerning the statistical confidence that can be attributed to extreme values severely limits the effectiveness of coastal protection and flood risk management and can lead to expensive and inappropriate decisions being taken.

In much coastal and ocean engineering design, it is important to consider the joint occurrence of combined conditions, such as high water levels and large waves. As an example, semi-empirical techniques were proposed by Hawkes et al. (2002) for estimating design conditions for coastal flood defences. These determine the degree of dependence between two variables and then apply an intuitive method to estimate joint extremes. This type of approach has been put onto a rigorous footing (see, e.g. Coles and Tawn, 1994). More recently, Heffernan and Tawn (2004) have presented a conditional approach for multivariate extremes. However, such techniques are slow in being taken up in engineering practice, due to reasons including: the cost of obtaining sufficient data for more complex methods; the cost of staff training on techniques that are not "industry standard"; the general success of simpler univariate methods coupled with engineering judgement. Nevertheless, there is a growing requirement amongst informed clients and practitioners to be able to quantify the uncertainty associated with estimated design conditions. For example, Todd and Walton (2000) investigated the storm surge at Sandy Hook, New Jersey. It was suggested that considerable concentration should be paid to the fitting a model to the upper tail of the data. Nevertheless, little attempt was made to estimate the quality of this fit. One of the most important parameters in coastal engineering design is the return period of wave heights. In this paper, we concentrate on univariate extreme value models, and methods for quantifying the associated uncertainty in estimates of extreme values of significant wave height for particular return periods.

* Corresponding author. Present address: DeepSea Engineering & Management Ltd., 39A East Street, Epsom, Surrey KT17 1BL, UK. Tel.: +44 1372739397; fax: +44 1372741497.

E-mail addresses: yli@deepsea-eng.com (Y. Li), dave.simmonds@plymouth.ac.uk (D. Simmonds), dominic.reeve@plymouth.ac.uk (D. Reeve).

0029-8018/$ - see front matter © 2008 Elsevier Ltd. All rights reserved.
doi:10.1016/j.oceaneng.2008.02.009

Given several years of wave height data, the typical approach for calculating this parameter is to extract annual maximum wave heights and then to fit a candidate cumulative density function (CDF) to the data. Suitable CDFs vary in form and typically have two or three parameters. Once the CDF parameters have been optimised against the wave statistics, the CDFs are used to estimate the return period values for the wave heights in question. A recent summary of the uncertainty of model fitting and the confidence intervals of wave parameters using Monte Carlo simulation was presented by Goda (2000). Subsequently, Goda (2004) investigated the behaviour of the upper tail of the CDF in terms of a spread parameter defined as the ratio of the 50-year return period to the 10-year wave heights. In both of these investigations, it was initially necessary to fix the shape parameter in recognition of the fact that a sample of a few dozen to one hundred data points is rather small.

Whilst closed-form (analytical) formulae for standard errors are available in particular cases, this is not guaranteed when considering an error norm such as used in this paper. In this case, the standard errors must be determined by alternative means, such as directly from bootstrap samples. Additionally, the finite nature of the sample tends to give rise to a further uncertainty in the parameters. It might be thought that if such a dataset is regularly sampled over short intervals then monthly, weekly or even daily maxima could in principle be extracted to increase the number of observations. Whilst this is possible in some circumstances, the problem with this approach in general is that the observations must generally be independently and identically distributed (IID) for the extreme value CDFs to be appropriate. However, O'Brien (1987) argued that the general extreme value (GEV) distribution could also be an appropriate model for the distribution of block maxima for stationary dependent sequences, provided there is only short range dependence. Often periods of storminess may extend over days and weeks and thus the maxima at weekly or monthly scales may become correlated (Hawkes et al., 2002).

A more promising approach for defining a reliable measure of goodness-of-fit is to resample the existing data, that is, create new samples from the original sample. Resampling enables an investigation of the stability or uncertainty of the fitted CDF to be performed and can also provide an estimate of the uncertainty in the extreme values. Intuitively, this may seem improbable and it must be noted that resampling does not create new information—it simply allows a fuller exploration of the statistics of the original data. By resampling with omission and/or repetition of the original sample, the stability of the statistical parameters derived from the observations can be determined. Both Rossouw (1988) and Michael and Hensley (1990) have reported attempts of using resampling techniques for estimating extreme wave heights.

The bootstrap resampling and the jackknife resampling are two such techniques of particular interest here. The bootstrap resampling is a computer-intensive method that has been available for more than 20 years (Efron, 1979), but has only recently become more widely used with advances in computing power. The jackknife resampling is even an older technique for estimating the bias and standard error of an estimate by resampling. The jackknife resampling often provides a good approximation to the bootstrap resampling; however, it is known that jackknife resampling fails in certain situations (e.g. Efron and Tibshirani, 1993).

In what follows, we begin by briefly reviewing the most common CDFs that are applied to extreme probability estimates of wave heights. In particular, we discuss the Weibull, the GEV, and the Gumbel distributions. We then discuss the two resampling techniques, which are introduced for the purpose of evaluating the CDF models fitted to limited samples. Having explained this use of resampling, we show how an improved estimate of the error norm of a CDF can be constructed from resampled data. The error norm is used to choose the most appropriate probability model for extreme wave heights observed at three different locations: Alghero, Italy; Ebro Delta, Spain; and Duck, USA. The error norm of the resampled data for each of the functions is evaluated and the extreme values at given return periods are then compared. The sensitivity of the CDF parameters is also examined.

2. Methodology

2.1. Cumulative density functions

For completeness, we briefly review the properties of the most common CDF forms employed first. The Weibull distribution is an extremely important distribution for characterising the probabilistic behaviour of a large number of real world phenomena. In particular, it has been used for coastal engineering problems and also for assessing reliability and life times of "products" in general. The three-parameter Weibull distribution function, abbreviated henceforth as Weibull (III), is given by

$$F(x) = 1 - \exp\left[-\left(\frac{x-\mu}{\sigma}\right)^{\rho}\right], \tag{1}$$

where the three parameters μ, σ, and ρ are the location, scale, and shape parameter. These control the locality, the spread (scale) and the asymmetry (shape) of the distribution, respectively. The tail behaviour is influenced by the shape parameter. The parameters are limited such that $x \geqslant \mu$, $\sigma > 0$, $\rho > 0$. The Weibull distribution is a special case of the GEV. In this work, we follow the method proposed by Qiao and Tsokos (1995) to estimate the maximum likelihood parameters of the Weibull (III) distribution. When $\mu = 0$, Eq. (1) is reduced to the two-parameter Weibull distribution, abbreviated as Weibull (II). This is more often and easily used as the Weibull (III) requires the solving of a system of nonlinear equations for estimating the parameters.

The GEV distribution was introduced by Jenkinson (1955). The GEV is capable of describing all three asymptotic behaviours identified by Fisher and Tippett (1928) and has been widely used in the analysis of extreme distribution because it has offers great flexibility with three free parameters:

$$F(x) = \exp\left[\left(1 - \kappa\left(\frac{x-\eta}{\lambda}\right)\right)^{1/k}\right], \tag{2}$$

where η, κ, and λ are the location, shape, and scale parameters. These are limited such that

$$1 - \kappa\left(\frac{x-\eta}{\lambda}\right) > 0, \quad \kappa \neq 0.$$

The Gumbel distribution (Gumbel, 1954), also known as Extreme type (I), is a special case of the GEV in the limit $\kappa \to 0$, given by

$$F(x) = \exp\left[-\exp\left(-\frac{x-\eta}{\lambda}\right)\right]. \tag{3}$$

2.2. Resampling techniques

We introduce the use of resampling technique to estimate the expectation of the error norm. In this context all the sample data are regarded as realizations of an IID stationary random process. This can be validated by examining the auto-correlation of the time series of data.