Available online at www.sciencedirect.com
ScienceDirect
Ocean Engineering 34 (2007) 1631–1649
OCEAN ENGINEERING
www.elsevier.com/locate/oceaneng

Wave-height distributions and nonlinear effects

M. Aziz Tayfun[a,*], Francesco Fedele[b]

[a]Civil Engineering Department, College of Engineering and Petroleum, Kuwait University, P.O. Box 5969, Safat 13060, Kuwait
[b]School of Civil & Environmental Engineering, Georgia Institute of Technology, 210 Technology Circle, Savannah, GA 31407, USA

Received 19 May 2006; accepted 1 November 2006
Available online 12 February 2007

Abstract

Theoretical distributions proposed for describing the crest-to-trough heights of linear waves are reviewed briefly. To explore the effects of nonlinearities, these are generalized to second-order waves, utilizing quasi-deterministic results on the expected shape of large waves. The efficacy of Gram–Charlier models in describing the effects of third-order nonlinearities on the distributions of wave heights, crests and troughs are examined in detail. All models and a fifth-order Stokes–Rayleigh type model recently proposed are compared with linear and nonlinear waves simulated from the JONSWAP spectrum representative of long-crested extreme seas, and also with oceanic data gathered in the North Sea. Uncertainties arising from the variability of probability estimates derived from sample populations of limited size are considered. Ultimately, the comparisons show that nonlinearities do not have any discernable effect on the crest-to-trough heights of oceanic waves. Most of the linear models considered yield similar and reasonable predictions of the observed data trends. Gram–Charlier type distributions seem neither effective nor particularly useful in describing the statistics of large wave heights or crests under oceanic conditions. However, they do surprisingly well in predicting unusually large wave heights and crests observed in some 2D wave-flume experiments and 3D numerical simulations of long-crested narrow-band random waves.

Keywords: Cumulants; Extreme values; Gram–Charlier series; Nonlinear waves; Probability distributions; Sampling variability; Wave crests; Wave envelopes and phases; Wave heights; Wave interactions; Wave troughs

1. Introduction

Current interest in the mechanics and statistics of large waves necessitates a re-examination of various theoretical forms for describing the distributions of wave heights and crests (see e.g. Haver and Andersen, 2000; Stansell, 2004; Walker et al., 2004; Fedele and Arena, 2003, 2005; Janssen, 2005; Guedes Soares and Pascoal, 2005; Socquet-Juglard, 2005; Socquet-Juglard et al., 2005; Fedele, 2006; Onorato et al., 2004, 2005, 2006; Tayfun, 2006). Over the years, a variety of numerical, empirical and analytic wave height and crest models have been proposed. Most of these have been reviewed and compared previously (Forristall, 1984, 2000; Tayfun, 1990a, 2006). This study will first focus on an initial short list of just three analytic crest-to-trough wave-height models due to Tayfun (1981, 1990a), Naess (1985) and Boccotti (1989). Of these, Naess' model (N) has received wide popularity. This is justifiably so due to its simple functional form although previous comparisons (Tayfun, 1990a) and those to be presented here indicate that N underestimates the observed wave heights slightly. Tayfun's model (T) for large wave heights is consistently more accurate than N. But, it is totally ignored apparently because its functional form is more complex and thus less amenable to analytical and/or practical applications than N. However, Boccotti's model (B) is just about as simple as N, but has not received the attention it probably deserves either. Thus, one of the present objectives is to review these models briefly, including two obvious and simple approximations that follow from T. The variability of probability estimates derived from sample populations of limited size and its relevance in interpreting the nature of exceptionally large waves are also considered. Subsequently, all the linear

*Corresponding author. Tel.: +965 622 7814; fax: +965 481 7524
E-mail address: aziztayfun@usa.net (M.A. Tayfun).

0029-8018/$ - see front matter
doi:10.1016/j.oceaneng.2006.11.006

models are compared with simulated linear waves and oceanic data gathered during two severe storms in the northern North Sea.

Since large waves are nonlinear, the linear models considered are subsequently modified to include first the effects of second-order nonlinearities, using the extension of Boccotti's (1989, 2000) linear quasi-deterministic theory to second-order waves by Fedele and Arena (2005). The resulting second-order models, a fifth-order Stokes–Rayleigh model recently proposed by Dawson (2004) and a second-order version that follows from it are then compared with simulated nonlinear waves representative of extreme seas and also with the same North Sea data.

Nearly all past oceanic observations as well as the present measurements gathered during two exceptionally severe storms clearly indicate that wave heights are not affected by nonlinearities. However, some recent analyses by Janssen (2003, 2005), Mori and Janssen (2006), and Onorato et al. (2004, 2005, 2006) based on the nonlinear Schrödinger (NLS) equation and wave-flume experiments show that occurrences of so-called 'freak' waves and wave heights considerably larger than those typically predicted with the conventional probability laws can be explained in terms of third-order nonlinear interactions. Mori and Janssen (2006) also contend that the distribution of such wave heights is described by a modified form of the Gram–Charlier (GC) series dependent solely on the kurtosis of surface elevations. Dysthe et al. (2005), Socquet-Juglard (2005) and Socquet-Juglard et al. (2005) explore the effects of third-order nonlinearities further with a series of intriguing 3D numerical simulations based on the Dysthe equation, a higher-order form of the NLS equation modified for directional waves with large steepness and broader spectra (Dysthe, 1979; Trulsen and Dysthe, 1996; Trulsen and Stansberg, 2001; Dysthe et al., 2003). These confirm that an increased density of unusually large waves does in fact appear in nearly 2D or long-crested waves initially characterized by relatively narrow-band spectra. This tends to occur in the absence of dissipation and surface stresses, and as spectra change relatively rapidly due to modulational instabilities toward an equilibrium range proportional to ω^{-4} over high frequencies. However, the simulations representative of the more realistic short-crested waves also show clearly that similar spectral changes do not cause any discernable aberrations, and the statistical characteristics of the free surface elevations, wave heights and crests are described surprisingly well with the presently available linear and second-order probabilistic models.

The possibility that third-order nonlinearities can modify the statistical structure of surface waves dramatically under certain conditions also necessitates a re-examination of the efficacy of GC type approximations in describing the distributions of large wave heights, crests and troughs. Thus, GC models appropriate to third-order waves are considered, drawing on the formulations devised previously in Tayfun and Lo (1990) and Tayfun (1994, 2006), and extending these to wave crests and troughs. All the resulting theoretical expressions are then compared with the North Sea data, and also with some 3D simulations from Socquet-Juglard et al. (2005) and 2D wave-flume data from Onorato et al. (2004, 2005, 2006).

2. Linear waves

2.1. Definitions

Consider linear deep-water waves, and let S represent the surface spectral density as a function of angular frequency ω. Denoting the ordinary moments of S by $m_j\ (j = 0, 1, \ldots)$, $\sigma \equiv m_0^{1/2}$ corresponds to the root-mean-square (r.m.s.) surface elevation, and $\omega_m = m_1/m_0$, $T_m = 2\pi/\omega_m$, $v = \sqrt{(m_0 m_2/m_1^2) - 1}$ and $\omega_0 = \sqrt{m_2/m_0} = \omega_m\sqrt{1+v^2}$ define the spectral average frequency, associated wave period, spectral bandwidth and the mean zero up-crossing frequency, respectively. Further, the r.m.s. surface gradient is given by $\mu \equiv m_4^{1/2}/g$ with $g \approx 9.8\ \mathrm{m/s^2}$. In general, $\mu^2 \ll 1$ since $\mu \approx O(10^{-1})$ at most.

Next, let η and $\dot{\eta} = \partial\eta/\partial t$ represent, respectively, the surface elevation from the mean sea level and its time derivative at a fixed point as a function of time t. Scaling η with σ and $\dot{\eta}$ with $m_2^{1/2} = \sigma\omega_0$ allows their normalized autocorrelation kernels to be expressed as

$$\rho(\tau) = \langle \eta(t)\eta(t+\tau)\rangle = m_0^{-1}\int_0^\infty S(\omega)\cos(\omega\tau)\,\mathrm{d}\omega, \tag{1}$$

$$\rho''(\tau) = \langle \dot{\eta}(t)\dot{\eta}(t+\tau)\rangle = -m_2^{-1}\int_0^\infty \omega^2 S(\omega)\cos(\omega\tau)\,\mathrm{d}\omega, \tag{2}$$

where $\rho'' = \mathrm{d}^2\rho/\mathrm{d}\tau^2$. The upper (+) and lower (−) envelopes of ρ are then given by

$$\pm r(\tau) = \pm\sqrt{\rho^2 + \hat{\rho}^2}, \tag{3}$$

where

$$\hat{\rho}(\tau) = \langle \eta(t)\hat{\eta}(t+\tau)\rangle = m_0^{-1}\int_0^\infty S(\omega)\sin(\omega\tau)\,\mathrm{d}\omega, \tag{4}$$

and $\hat{\eta}$, $\hat{\rho} \equiv$ Hilbert transforms of η and ρ, respectively.

Now, define the dimensionless parameters

$$r_m = r(\tau_m), \quad a = \rho(\tau^*), \quad b = \rho''(\tau^*), \tag{5}$$

where $\tau_m = T_m/2$ for simplicity, and $\tau^* \equiv$ the time lag at which the first minimum of ρ occurs (Boccotti, 1989, 2000). These parameters can all be estimated either from a time series of surface elevations or, somewhat more accurately and just as simply from the associated frequency spectrum. For example, consider

$$S(\omega) = \frac{m_0}{\omega_p}u^{-4}\exp(-1.25u^{-4})\gamma^{g(u)}, \tag{6}$$

where $\omega_p \equiv$ spectral-peak frequency, $\gamma \equiv$ peak-enhancement coefficient, $u = \omega/\omega_p$, and $g(u) \equiv$ standard JONSWAP