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A higher-order σ-coordinate non-hydrostatic model for nonlinear surface waves

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Abstract

A higher-order non-hydrostatic model in a σ-coordinate system is developed. The model uses an implicit finite difference scheme on a staggered grid to simultaneously solve the unsteady Navier–Stokes equations (NSE) with the free-surface boundary conditions. An integral method is applied to resolve the top-layer non-hydrostatic pressure, allowing for accurately resolving free-surface wave propagation. In contrast to the previous work, a higher-order spatial discretization is utilized to approximate the large horizontal pressure gradient due to steep surface waves or rapidly varying topographies. An efficient direct solver is developed to solve the resulting block hepta-diagonal matrix system. Accuracy of the new model is validated by linear and nonlinear standing waves and progressive waves. The model is then used to examine freak (extreme) waves. Features of downshifting focusing location and wave asymmetry characteristics are predicted on the temporal and spatial domains of a freak wave.

Keywords: σ-coordinate; Non-hydrostatic pressure; Free-surface waves; Freak waves

1. Introduction

Accurate prediction of wave climate in deep-water and shallow-water is a prime subject in physical oceanography and coastal engineering (Mei and Liu, 1993; Battjes, 2006; Cavaleri, 2006). Wave models can be generally divided into two classes: (i) phase-resolving models that predict both the amplitude and phase of individual waves and (ii) phase-averaging methods which predict average or integral properties of the wave field such as significant wave height or peak wave period. Over the years, a good progress on phase-averaging modeling has been made (Hasselmann, 1974; Komen et al., 1984; Booij et al., 1999). While there are continuous improvements in the formulation of energy source terms including wind input formulation (Tolman and Chalikov, 1996), nonlinear wave–wave interactions (Tanaka, 2001; Van Vledder and Bottema, 2002; Janssen, 2003) and wave dissipations (Henrique et al., 2003; Yao and Wu, 2004), phase-averaging models become questionable for simulating non-stationary and nonlinear waves or rapidly varying wave field like freak waves (Cavaleri, 2006). In contrast phase-resolving models are capable of simulating complicated nonlinear wave–wave interactions. Many efforts have been paid to develop a unified model that can predict wave transformation including shoaling, refraction, diffraction, and reflection from deep-water to shallow-water zone (Liu and Losada, 2002). A good review of this topic can refer to Battjes (1994) and Choi and Wu (2006). Nevertheless, continuously developing efficient models for resolving nonlinear and dispersive deep-water waves or nonlinear shallow-water waves over irregular, steep topography remains a challenging task.

In recent years, many non-hydrostatic models that track the free-surface motion using a single-valued function of the horizontal plane have been developed (Mahadevan et al., 1996; Casulli and Stelling, 1998; Mayer et al., 1998; Casulli, 1999; Namin et al., 2001; Li and Fleming, 2001; Kocyigit et al., 2002; Lin and Li, 2002, Stelling and Zijlema, 2003; Chen, 2003; Yuan and Wu, 2004a, b;

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Bradford, 2005; Walters, 2005; Lee et al., 2006; Choi and Wu, 2006). Unlike hydrostatic models that neglect the vertical acceleration or the depth-integrated formulation like Boussinesq-type models (Peregrine, 1967) that are more applicable to relatively shallow-water conditions, models based upon a non-hydrostatic pressure distribution can effectively simulate both coastal and deep-water surface wave motions (Lin and Liu, 1998; Stelling and Zijlema, 2003; Yuan and Wu, 2004a, b; Walters, 2005). In addition the effects of currents on wave evolutions (Yuan and Wu, 2006) and wave-induced current modulations (Groeneweg and Battjes, 2003) can be easily included in non-hydrostatic models. While the advantages of non-hydrostatic models are apparent, applications of models in simulating nonlinear and dispersive waves over uneven topographies are still limited.

Two possible reasons hinder the applications of non-hydrostatic modeling. First, to capture the rapidly moving nonlinear waves, methods like the arbitrary Lagrangian–Eulerian method (Zhou and Stansby, 1999; Hodges and Street, 1999), the marker and cell method (Harlow and Welch, 1965; Armenio and LaRocca, 1996), the volume of fluid method (Hirt and Nichols, 1981), and the level-set method (Iafrati and Campana, 2003) have been successfully implemented in NSE-based models. Nevertheless, high computational expense of these methods usually limits practical applications of models. Second, non-hydrostatic models based upon a so-called moving top-layer method under a Cartesian coordinate system can efficiently handle high steepness waves under sheared flow (Yuan and Wu, 2006) or turbulence (Wu and Yuan, 2006) but have an overshooting issue in modeling nonlinear waves in which the wave trough is below the bottom of the top vertical layer (Casulli and Stelling, 1998; Casulli, 1999; Chen, 2003; Stelling and Zijlema, 2003; Yuan and Wu, 2004b). A so-called top-down resolving free-surface method (Yuan and Wu, 2006) only serves as a posterior to set up the thickness of the top vertical layer. On the other hand, a σ-coordinate system naturally alleviates the overshooting issue by ideally mapping the free-surface wave motion and irregular bottom into a fixed rectangular prism (Namin et al., 2001; Li and Fleming, 2001; Kocyigit et al., 2002; Lin and Li, 2002; Yuan and Wu, 2004a; Bradford, 2005; Lee et al., 2006). Interestingly, this type of models has been applied to simulate mainly linear small-amplitude waves. To dates very few studies have yet used non-hydrostatic σ-coordinate models for accurately modeling nonlinear (or steep) and short (or deep-water) dispersive waves.

Issues of σ-transformation at steep bottom topographies may explain some grid deterioration concerns at steep free-surface waves. In the past issues of using the σ-coordinate-based models to simulate flows over steep bottom topography have been well documented (Haney, 1991; Stelling and van Kester, 1994; Mellor et al., 1994). Specifically, small pressure gradients can be resulted from two relatively large terms of opposite sign, yielding a relative large error in the pressure gradient that induces artificial flows. To reduce errors of σ-coordinate hydrostatic models near the steep topography, three approaches are (i) to bring certain symmetries of the continuous forms in to the discrete levels to ensure cancellations of these terms (Mellor et al., 1994; Song, 1998); (ii) to change the grid from a σ-grid to a z-level grid before the calculation of the horizontal pressure gradient (Stelling and van Kester, 1994), and (iii) to increase numerical accuracy (McCalpin, 1994; Chu and Fan, 2001). To the best of the authors' knowledge, all these approaches have not been applied to non-hydrostatic σ-coordinate models for simulating steep free-surface waves.

In this paper, a high-order numerical scheme is applied to discretize the horizontal pressure gradient term in the non-hydrostatic model (Yuan and Wu, 2004a), which is based upon an implicit finite difference staggered scheme to simultaneously solve the unsteady NSE and the free-surface boundary conditions. Under a σ-coordinate system, an improved integral method (Yuan and Wu, 2006; Choi and Wu, 2006) is applied to resolve the top-layer non-hydrostatic pressure, allowing for accurately resolving surface wave propagation. An efficient direct solver to solve a resulting block hepta-diagonal matrix system is developed. In the following sections, governing equations and boundary conditions in the σ-coordinates are given first. Numerical method is presented next. Accuracy of the new model is validated by linear and nonlinear standing waves and progressive waves. The model is finally used to examine the fast evolution of freak (extreme) waves. Characteristics of freak wave are examined and discussed.

2. Governing equations and boundary conditions

For incompressible flows, the governing equations are the Navier–Stokes equations (NSE), which describe the conservation of mass and momentum. Through the transformation $t = t^*$, $x = x^*$, and $\sigma = z^* - \eta/h + \eta = z^* - \eta(x,t)/H(x,t)$, where $H(x,t)$ is the total water depth, the Cartesian x^*–z^*–t^* coordinate is mapped into a x–σ–t coordinate. In other words, the transformation maps a time-dependent physical domain, i.e., nonlinear waves over irregular bottoms, into a stationary rectangular σ domain, resulting in $\sigma = -1$ at the bottom and $\sigma = 0$ at the free surface, as shown in Fig. 1.

On a two-dimensional vertical plane, the continuity and momentum equations can be written in primitive variables under a σ-coordinate system as

$$\frac{\partial u}{\partial x} + \frac{\partial u}{\partial \sigma}\frac{\partial \sigma}{\partial x^*} + \frac{1}{H}\frac{\partial w}{\partial \sigma} = 0, \tag{1}$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + w_\sigma \frac{\partial u}{\partial \sigma} = -\left(\frac{\partial P}{\partial x} + \frac{\partial P}{\partial \sigma}\frac{\partial \sigma}{\partial x^*}\right) + \nu\nabla_\sigma^2 u, \tag{2}$$

$$\frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + w_\sigma \frac{\partial w}{\partial \sigma} = -\frac{1}{H}\frac{\partial P}{\partial \sigma} + \nu\nabla_\sigma^2 w - g, \tag{3}$$