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A combined numerical–empirical method to calculate finite-time Lyapunov exponents from experimental time series with application to vessel capsizing

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Abstract

This paper presents a method to calculate finite-time Lyapunov exponents (FTLEs) for experimental time series using numerical simulation to approximate the local Jacobian of the system at each time step. This combined numerical–experimental approach to the calculation of FTLE is applicable to any physical system which can be numerically approximated. By way of example, the method is applied to the problem of vessel capsize.

Keywords: Finite-time Lyapunov exponents; Capsize; Jacobian

1. Introduction

The strong sensitivity of vessel capsizing to initial conditions has been a subject of research for decades (Paulling and Rosenberg, 1959; Thompson, 1997; Spyrou and Thompson, 2000; Lee et al., 2006). This sensitivity is an inherent sign of a chaotic system (Theiler, 1990), therefore an intuitive approach to the quantitative study of

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capsize is to employ Lyapunov exponents. The Lyapunov exponent is a measure of the rate of convergence or divergence of nearby trajectories with positive values indicating exponential divergence and chaos. However, Lyapunov exponents are by definition an asymptotic parameter, whereas capsize is a finite-time phenomena. Therefore, finite-time Lyapunov exponents (FTLE) must be used to investigate behaviors leading to capsize. To yield insight applicable to realistic vessels, a feasible and physical method to calculate FTLE from experimental time series in combination with a simplified numerical model is presented.

The use of Lyapunov exponents to study capsize has been touched upon in the literature for both naval architecture and nonlinear dynamics. In recent years, the asymptotic Lyapunov exponent has been calculated from equations of motion for the mooring problem (Papoulias, 1987), single-degree-of-freedom capsize models (Falzarano, 1990; Murashige and Aihara, 1998a,b; Murashige et al., 2000; Arnold et al., 2003), and works studying the effects of rudder angle while surf-riding as it leads to capsize (Spyrou, 1996). Additionally, the authors conducted a study of the use of Lyapunov exponents to investigate large amplitude vessel roll motions in beam seas for a multi-degree of freedom numerical model in comparison to experimental results (McCue, 2004; McCue and Troesch, 2004). Based upon the results of this study, it was shown that the Lyapunov exponent can be used as a validation tool for large amplitude roll motion simulators. Through calculation of similar maximal Lyapunov exponent for experimental runs and numerically simulated runs, one can conclude that the numerical simulator likely captures the relevant, multi-dimensional physics of the problem. However, since the Lyapunov exponent is defined in the limit as time approaches infinity, it is ineffective for the study of the finite-time phenomena of capsize. This serves as the motivation for the present work in which capsize is studied using the finite-time measure given by the FTLE.

In order to be of use on-board a vessel and to make a sizeable improvement in safety, it is necessary to compute FTLEs from actual vessel time series in real time. While it is not impossible to approximate FTLEs through statistical methods (Lu, 1997; Lu and Smith, 1997) or approaches derived from those used for Lyapunov exponents (Wolff, 1992; Yao and Tong, 1994; Sano and Sawada, 1985), it is preferable for this research that the Jacobian approximation be calculated rapidly and in such a manner as to be physically intuitive. For a system such as the capsize model discussed herein, a numerical approximation for the Jacobian is readily available via a simulation tool. Therefore, to estimate the FTLEs for the experimental time series, a combined numerical–experimental approach is used.

Rather than using statistical or dimensionally limited methods to approximate the Jacobian of the system in time, a validated numerical simulator can be used to model the Jacobian in an incremental manner. For example, if given a time series from experimental data, and a numerical simulator capable of accurately integrating the equations of motion for an approximation of the system, one can use the simulator in a stepwise manner to estimate the Jacobian about each point in the experimental time series. For the system discussed in this paper, six-state variables are recorded in 1/30th of a second increments. Reading into the numerical simulator a row of experimental data containing these six-state variables, and integrating the linearized