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Modification of the damping function in the k–ε model to analyse oscillatory boundary layers

Ahmad Sana[a,*], Abdul Razzaq Ghumman[b], Hitoshi Tanaka[c]

[a]Department of Civil and Architectural Engineering, Sultan Qaboos University, P.O. Box 33, AL-KHOD, Postal Code 123, Sultanate of Oman
[b]Faculty of Civil Engineering, University of Engineering and Technology, Taxila, Pakistan
[c]Department of Civil Engineering, Tohoku University, Sendai, Japan

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Abstract

A simple relationship has been developed between the wall coordinate y^+ and Kolmogorov's length scale using direct numerical simulation (DNS) data for a steady boundary layer. This relationship is then utilized to modify two popular versions of low Reynolds number k–ε model. The modified models are used to analyse a transitional oscillatory boundary layer. A detailed comparison has been made by virtue of velocity profile, turbulent kinetic energy, Reynolds stress and wall shear stress with the available DNS data. It is observed that the low Reynolds number models used in the present study can predict the boundary layer properties in an excellent manner.

Keywords: Turbulence model; Oscillatory boundary layer; Shear stress; k–ε model

1. Introduction

A number of practical situations in fluid mechanics require an understanding of oscillatory boundary layers and turbulence associated with them. Many experimental and analytical studies have been carried out on this topic. Comprehensive reviews of these studies have been published by Sleath (1990) and Soulsby et al. (1993). The idea of using turbulence models to tackle this phenomenon is relatively new. With the availability of excellent computing facilities at affordable costs, this option is gaining more popularity among the researchers and the practicing engineers. The benefit of using a good turbulence model is that it produces detailed boundary layer properties at a reasonable cost within short time. For a turbulence model to be good, an essential requirement is computational economy with reasonable accuracy. The present study deals with the application of low Reynolds number k–ε models to an oscillatory boundary layer. The term *low Reynolds number* implies that this model is applicable over the whole cross-stream dimension including the low Reynolds number region (viscous sublayer).

The low Reynolds number k–ε model was originally developed by Jones and Launder (1972) and then various modifications were proposed to widen its scope of applications or improve the predictive ability of this model. Most of these modifications proved to be ad hoc in nature after applying these modified versions to the cases other than they were developed for. Some of these models were applied to oscillatory boundary layers as well. Patel et al. (1985) reviewed some of the versions of two-equation models in relation to steady flow phenomena. In this study, the two-equation models were reviewed for their correct near-wall behaviour as well. Moreover, a valuable experimental data gathered from various sources were provided that proved to be helpful in developing new models or modifying them. A number of modified versions of two-equation turbulence models were published based on the findings of Patel et al. (1985).

Although two-equation models were mainly developed for steady boundary layers, their application to oscillatory

*Corresponding author. Tel.: +968 2414 2524; fax: +968 2414 3416.
E-mail addresses: sana@squ.edu.om (A. Sana), abdulrazzaq@uettaxila.edu.pk (A.R. Ghumman), tanaka@tsunami2.civil.tohoku.ac.jp (H. Tanaka).

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boundary layers has been successful too (Thais et al., 1999; Cotton and Stansby, 2000; Foti and Scandura, 2004; Shen et al., 2004; Sue et al., 2005). Tanaka and Sana (1994) reviewed some of the older versions with reference to oscillatory boundary layer properties by using the available experimental data. As a result of this study it was found that the original model by Jones and Launder (1972) performed better than the rest of the models tested, especially considering the transitional properties of oscillatory boundary layers. Sana and Tanaka (2000) concluded after reviewing the original and four newer versions with reference to the available direct numerical simulation (DNS) data for oscillatory boundary layers that the turbulence models proposed by Myong and Kasagi (1990) and Nagano and Tagawa (1990) showed better agreement with the DNS data by virtue of the shape of the turbulent kinetic energy profile. But it was noted that the expressions for the damping function used in many of the newer versions of low Reynolds number k–ε models involve the wall coordinates y^+ $(= yu_f/\nu$, $y =$ cross-stream distance, $u_f =$ shear velocity and $\nu =$ kinematic viscosity). In case of oscillatory flow, the bottom shear stress goes to zero twice in a wave cycle and at that time the damping function based on y^+ becomes zero in the whole cross-stream dimension, which is physically incorrect.

In the present study, a simple relationship has been developed and then utilized to transform wall coordinates to another suitable variable based on the DNS data for steady boundary layers. The damping functions of two of the popular versions proposed by Myong and Kasagi (1990) and Nagano and Tagawa (1990) are thus modified and tested against the DNS data for oscillatory boundary layers along with the original model by Jones and Launder (1972).

2. Methodology

2.1. Governing equations of k–ε model

Using the eddy viscosity concept, the equation of motion, for one-dimensional oscillatory boundary layer, may be written as

$$\frac{\partial u}{\partial t} = \frac{\partial U}{\partial t} + \frac{\partial}{\partial y}\left\{(\nu + \nu_t)\frac{\partial u}{\partial y}\right\}, \tag{1}$$

where u is the velocity in x-direction, U the free-stream velocity, t the time, y the cross-stream dimension and ν_t the eddy viscosity. According to the general form of low Reynolds number k–ε model, the eddy viscosity is expressed as

$$\nu_t = C_\mu f_\mu \frac{k^2}{\tilde{\varepsilon}}, \tag{2}$$

where C_μ is a constant (= 0.09 for the models considered here) and f_μ is the damping function. The turbulent kinetic energy, k, transport equation is

$$\frac{\partial k}{\partial t} = \frac{\partial}{\partial y}\left\{\left(\nu + \frac{\nu_t}{\sigma_k}\right)\frac{\partial k}{\partial y}\right\} + \nu_t\left(\frac{\partial u}{\partial y}\right)^2 - \tilde{\varepsilon} - D. \tag{3}$$

And the transport equation of turbulent kinetic energy dissipation rate, $\tilde{\varepsilon}$, is

$$\frac{\partial \tilde{\varepsilon}}{\partial t} = \frac{\partial}{\partial y}\left\{\left(\nu + \frac{\nu_t}{\sigma_\varepsilon}\right)\frac{\partial \tilde{\varepsilon}}{\partial y}\right\} + C_1 f_1 \nu_t \frac{\tilde{\varepsilon}}{k}\left(\frac{\partial u}{\partial y}\right)^2 - C_2 f_2 \frac{\tilde{\varepsilon}^2}{k} + E. \tag{4}$$

Here, C_1, C_2, σ_k and σ_ε are model constants and f_1, f_2, D and E are model functions.

Jones and Launder (1972) (JL model) proposed an expression based on turbulence Reynolds number only, i.e. $f_\mu = \exp\{-2.5/(1 + R_t/50)\}$, $(R_t = k^2/(\tilde{\varepsilon}\nu))$. But, in some of the later versions, for example, Myong and Kasagi (1990) (MK model) and Nagano and Tagawa (1990) (NT model) used wall coordinate (y^+) in the expression for the damping function as follows:

MK model:

$$f_\mu = \left(1 + 3.45/\sqrt{R_t}\right)(1 - \exp(-y^+/70)). \tag{5}$$

NT model:

$$f_\mu = (1 + 4.1/R_t^{0.75})(1 - \exp(-y^+/26))^2. \tag{6}$$

In case of steady boundary layers, these models (MK and NT models) perform very well and sometimes better than the JL model, which proves that the wall distance should be somehow incorporated in the damping function. But in case of oscillatory boundary layers, the wall shear stress goes to zero twice in a wave cycle and results in the zero value of y^+ at those instants. From Eqs. (5) and (6) it may be readily noted that the damping function goes to zero at these instants leading to a zero eddy viscosity (from Eq. (2)). The experimental data for oscillatory boundary layers show that the turbulent kinetic energy is produced near the wall during acceleration phase and then spreads in the cross-stream direction during deceleration. Therefore, even when the wall shear stress is zero, the eddy viscosity is not zero over the whole cross-stream dimension. This physically incorrect behaviour of MK and NT models renders them to be unsuitable for not only oscillatory boundary layers but the boundary layers under adverse pressure gradient as well.

Abe et al. (1994) utilized Kolmogorov's length scale y^* $(= (\nu\varepsilon)^{1/4} y/\nu)$ based on the argument that in the close vicinity of the wall this length scale is very important due to its dependence on the dissipation rate of the turbulent kinetic energy ε. With the availability of DNS data for various types of boundary layers, it is now possible to develop the empirical relationships for different model parameters. In the present study, the DNS data by Kuroda et al. (1990) for steady boundary layer in a smooth channel is utilized to find the relationship between y^+ and y^*. By plotting these two variables it was observed that $y^+ \cong 1.65 y^*$ for $y^+ \leqslant 3$. Therefore, Eqs. (5) and (6) may be