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A successive elimination method for one-dimensional stock cutting problems in ship production

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Abstract

One-dimensional stock cutting problems can be encountered at the production stage of many areas of engineering as well as in shipbuilding and coastal structures. In this paper, a novel approach is proposed to solve the problem directly by using the cutting patterns obtained by the analytical methods at the mathematical modeling stage. By minimizing both the number of different cutting patterns and material waste, the proposed method is able to capture the ideal solution of the analytical methods. The main advantage of the method comes from the fact that an integer solution is guaranteed. However, in analytical methods it is not always possible to produce integer solutions and the linear programming algorithm must be run repeatedly to select integer solutions from the alternatives to get practical results. The proposed nesting algorithm is a low-cost and efficient tool. Minimizing the number of cutting patterns contributes to time and material savings. Also, by using this method trim loss is minimized and stock usage is maximized. The efficiency of the proposed method is demonstrated by extensive numerical results.

Keywords: Stock cutting; Part nesting; Linear programming; Heuristic approach

1. Introduction

In industrial cutting operations such as shipbuilding, stock material input is a very important criterion. Cutting plans must be prepared to obtain the required set of pieces from the available stock lengths. The primary objective is to minimize the number of used stock material or equivalently trim loss (wastage). Since switching between different patterns can be time-consuming and prone to setup errors, the number of cutting patterns contained in a cutting plan solution should be minimized.

The first attempts to solve the one-dimensional (1D) cutting stock problem (CSP) by analytical means were proposed by Gilmore and Gomory (1961, 1963, 1965). They initially determined the feasible cutting patterns which describe how many items of each type are cut from stock lengths (the pattern run-lengths). The solution was then achieved by using the mathematical model which was based on these patterns. According to Dyckhoff (1981, 1990), the classic 1D-CSP is classified as a one-dimensional problem with an unlimited supply of rolls of identical size and a set of orders that must be fulfilled. Waescher and Gau (1996) carried out extensive computational experiments with instances with an average demand per order width of 10 and 50, and concluded that the optimal integer solutions could be obtained in most cases. Furthermore, when the heuristics of Waescher and Gau (1996) and the heuristic of Stadtler (1990) are used in conjunction with each other, they solve almost every instance of the CSP to an optimum. In recent years, there have been several efforts to solve this problem by LP-based branch-and-bound with column generation (called branch-and-price) and by general-purpose Chvatal–Gomory cutting plans. Nevertheless, Vance et al. (1994), Vance (1998), Vanderbeck (1999, 2000) and Valerio de Carvalho (1999) recently presented some attempts at combining column generation and branch-and-bound, a framework that has also been

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successfully applied to other integer programming problems and is usually denoted as branch-and-price. They were able to solve exactly quite large instances of CSPs. Scheithauer et al. (2001) presented an exact solution approach for the 1D-CSP which is based on a combination of the cutting plane method and the column generation technique. Belov and Scheithauer (2002) proposed that a cutting plane approach combining Chvatal–Gomory cutting planes with column generation is generalized for the case of multiple stock lengths in the 1D-CSP. Umetani et al. (2003) proposed an approach based on metaheuristics, and incorporates an adaptive pattern generation technique. Johnston and Sadinlija (2004) created a new model which resolves the non-linearity in the 1D-CSP, between pattern variables and pattern run-lengths by a novel use of 0–1 variables. Belov and Scheithauer (2006) investigated the effect of a combination of approaches: the LP relaxation at each branch-and-price node strengthened by Chvatal–Gomory and Gomory mixed-integer cuts.

In this paper, a successive elimination method has been proposed to solve 1D-CSP. The cutting plans are achieved directly without the need to establish a mathematical model. This new and simple solution method is developed based on the algorithms derived from the studies of Dikili (1991, 2004). The main objective of this method is to reach the optimal integer solution while minimizing the number of different patterns contained in a solution. It is demonstrated with numerical applications that screenings of all alternative patterns to reach an integer result are no longer required in this method. Thus, the proposed successive elimination method has given the ideal solution obtained by the conventional approach.

In Section 2, the analytical structure of the proposed algorithm is discussed. In Section 3, computational results of test cases are presented and results of both the conventional and the proposed method are compared. Finally, in Section 4, concluding remarks are presented.

2. The developed methodology

The present method, which is developed by using conventional and the heuristic methods, involves the following steps:

Step 1: Determine the length of the stock material (L), lengths of n different parts to be cut from the stock material and the demand of each part (D_i).

Step 2: Sort parts with descending lengths: for $i=\{1,2,\ldots,n\}, \forall\, i,\, i+1 \in I$ and $L_i \geqslant L_{i+1}$ The parts to be included are: $R=\{(L_1, D_1), \ldots, (L_i, D_i), \ldots, (L_n, D_n)\}$.

Step 3: Form the feasible cutting patterns according to the demand quantity of each part.

Step 3.1: For each pattern, determine the trim loss (waste), the maximum number of stock material to be used considering the demands of each item and the total number of parts to be cut as shown in Table 1.

For $j=\{1,\ldots,K\}$ and $i=\{1,\ldots,n\}$, $j \in J$,

Where D_i is the demand quantity of the ith part, $V_{j,I}$ the quantity of the ith part in the jth pattern and K the pattern number.

The values of Waste, Stock Material (SM_i), Parts Used (PU_j) can be calculated using the following relationship:

$$\mathrm{Waste}_j = L - \sum_{i=1}^{n} V_{j,i} \cdot L_i,$$

$$\mathrm{SM}_j = \min(\lfloor D_i / V_{j,i} \rfloor),$$

$$\mathrm{PU}_j = \mathrm{SM}_j . \sum_{i=1}^{n} V_{j,i},$$

where V_{ji} values can be calculated using the following recursive relationship:

$$V_{j,i+1} = \left\lfloor (L - \sum_{p=1}^{i} V_{j,p} . L_p)/L_{i+1} \right\rfloor,$$

$$V_{1,1} = \lfloor L/L_1 \rfloor.$$

Step 3.2: Determine the best pattern according to the following priorities:
(i) Minimum waste, Min (Waste_j)
(ii) Maximum stock material usage, Max(SM_j)
(iii) Minimum total number of parts used, Min(PU_j)
(iv) Maximum use of large parts, Max(K).

Step 3.3: Determine and store the best cutting pattern.

Step 3.4: Reorganize the demand quantities for the next elimination procedure.

Step 3.5: Renew the demand quantities and eliminate the patterns with excessive amount of parts.

Step 4: Terminate the iteration process if the cutting plan solution contains the required set of pieces, else go to Step 3.2.

3. Computational results

In this section, typical cases for 1D-CSP have been presented. The problems have been solved by both

Table 1
Cutting patterns for elimination process

L_1	L_2	…	L_n	Waste	No. of stock material	Total number of parts to be cut
D_1	D_2	…	D_n			
$V_{1,1}$	$V_{1,2}$	…	$V_{1,n}$	Waste $_1$	SM_1	PU_1
$V_{2,1}$	$V_{2,2}$	…	$V_{2,n}$	Waste $_2$	SM_2	PU_2
…	…	…	…	…	…	…
$V_{K,1}$	$V_{K,2}$	…	$V_{K,n}$	Waste_K	SM_K	PU_K