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Ocean Engineering 34 (2007) 54–68

OCEAN ENGINEERING

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The discrete methods for free vibration analyses of an immersed beam carrying an eccentric tip mass with rotary inertia

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Received 14 August 2005; accepted 28 December 2005
Available online 18 April 2006

Abstract

In this paper, a beam without contact with water is called the "dry" beam and the one in contact with water is called the "wet" beam. For a partially (or completely) immersed uniform beam carrying an eccentric tip mass possessing rotary inertia, the conventional analytical (closed-form) solution is achieved by considering the inertial forces and moments of the tip mass and rotary inertia as the boundary conditions at the tip end of the beam. However, it has been found that the approximate solution for the last problem may be achieved by two techniques: Method 1 and Method 2. In Method 1, the basic concept is the same as the conventional analytical method; but in Method 2, the tip end of the beam is considered as a free end, while the inertial forces and moments induced by the tip mass and rotary inertia are considered as the external loads applied at the tip end of the beam. The main differences between the formulation of Method 1 and that of Method 2 are: In Method 1, the "normal" shapes of the "dry" beam are functions of the frequency-dependent boundary conditions but the external loads at the tip end are equal to zero; On the contrary, in Method 2, the "normal" mode shapes of the "dry" beam are determined based on the zero boundary conditions at the tip end of the beam but the external loads at the tip end due to the inertial effects of the tip mass and rotary inertia must be taken into consideration for the free vibration analysis of the "wet" beam. Numerical results reveal that the approximate solution obtained from Method 2 are very close to that from Method 1 if the tip mass moment of inertia is negligible. Besides, the two approximate solutions are also very close to the associated analytical (closed-form) solution or the finite element solution. In general, it is hoped that there exist several methods for tackling the same problem so that one may have more choices to incorporate with the specified cases. It is believed that the two approximate methods presented in this paper will be significant from this point of view.

Keywords: Dry beam; Wet beam; Tip mass; Tip mass moment of inertia; Normal mode shapes; Frequency-dependent boundary conditions; Approximate solution

1. Introduction

Since the dynamic characteristics of some structures such as towers, piles, tall buildings and robot arms, can be predicted with reasonable accuracy from an elastically (or fixed) supported beam carrying a tip mass with (or without) rotary inertia, a lot of researchers devoted themselves to the study of problems in this aspect. For the free vibration analysis of "uniform" beams with tip mass, the works of Laura et al. (1974, 1975); Rossi et al. (1993); Wu and Lin (1990); Wu and Huang (1995); Wu and Chou (1998a,b); Abramovich and Hamburger (1991, 1992); Uscilowska and Kolodziej (1998) and Oz (2003) are some of the pertinent literature and for that of "non-uniform" beams with tip mass, the works of Laura and Gutierrez (1986); Lee (1976); Mabie and Rogers (1974); Auciello (1996); Chang and Liu (1989) and Wu and Chen (2003) are the associated articles. Among the above-mentioned literature, only the most concerned references will be reviewed here. For example, Chang and Liu (1989) have studied the natural frequencies of immersed restrained column by means of the transfer matrix method. Their purpose is to study the influence

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0029-8018/$ - see front matter © 2006 Elsevier Ltd. All rights reserved.
doi:10.1016/j.oceaneng.2005.12.003

of the following parameters: taper ratio of the beam, magnitudes of tip mass, eccentricity and rotary inertia, axial load, and the stiffness of elastic-support translational and rotational springs. Although most of the problems concerned has been studied by Chang and Liu (1989), the approach that they used is a conventional "approximate" method. For this reason, Uscilowska and Kolodziej (1998) presented the analytical method to determine the "exact" lowest five natural frequencies and mode shapes of the uniform cantilever tower carrying an eccentric tip mass with mass moment of inertia. Recently, the same problem as that done by Uscilowska and Kolodziej (1998) is studied by Oz (2003), he uses the numerical results of the conventional finite element method (FEM) to check the analytical (exact) solutions.

In reality, the support condition of an offshore tower is elastic rather than completely fixed. Hence, it will be more reasonable to model the interactions between the tower and the soil by using a translational (helical) spring and a rotational spring. In other words, the actual support condition for the lower end of an offshore tower will be close to the condition between weakly elastic support and fixed support. In such situation, two types of towers should be considered, one is elastically supported and the other is fixed supported. Therefore, Wu and Hsu (2006) presented a unified approach to determine the "exact" lowest several natural frequencies and the associated mode shapes of the last two types of (partially or fully) immersed beam. However, since determination of the "exact" natural frequencies and mode shapes of an immersed column with "in-span" lumped masses is not yet obtained no matter whether the column is uniform or non-uniform, this paper attempts to present two approximate methods (Method 1 and Method 2) to solve the last problem. To this end, the orthogonal conditions required by the "normal" mode shapes of the current vibrating system are presented. It is noted that, for the elastically supported tower carrying an eccentric tip mass with rotary inertia studied in this paper, all the boundary conditions for the current vibrating system are not equal to zero and parts of them are "frequency-dependent", but this is not true for the vibrating systems studied by Wu and Lin (1990) and Wu and Chen (2003). For this reason, the orthogonal conditions for Method 1 are much more complicated than those for the works of Wu and Lin (1990) and Wu and Chen (2003). Besides, the exact method presented by Uscilowska and Kolodziej (1998) or Wu and Hsu (2006) is available only for the immersed column without carrying any "in-span" realistic lumped masses, but both Method 1 and Method 2 presented in this paper are available for the immersed column with (or without) any number of "in-span" lumped masses. Good agreement between the results of Method 1 and Method 2 and those of the exact method (or the finite element method) confirms the reliability of the presented theory.

2. Natural frequencies and mode shapes of the "dry" beam for Method 1

For a "dry" uniform Euler–Bernoulli beam (cf. Fig. 1), its equation of motion is given by (Meirovitch, (1967); Uscilowska and Kolodziej, (1998))

$$EIy''''(x,t) + \rho A\ddot{y}(x,t) = 0, \tag{1}$$

where E is Young's modulus, A is cross-sectional area, I is moment of inertia of area A, ρ is mass density of beam material, x is the axial coordinate with origin at the lower end of the beam, $y(x,t)$ is the lateral displacement at position x and time t. Besides, in Eq. (1), the primes (′) refer to the derivatives with respect to (w.r.t.) coordinate x and the overhead dots ($\cdot$) refer to those w.r.t. time t.

For free vibration of the beam, one has

$$y(x,t) = \bar{Y}(x)e^{i\bar{\Omega}t}, \tag{2}$$

where $\bar{Y}(x)$ denotes the amplitude function of $y(x,t)$, while $\bar{\Omega}$ is the natural frequency of the "dry" beam and $i = \sqrt{-1}$.

The substitution of Eq. (2) into Eq. (1) leads to

$$\bar{Y}''''(x) + \bar{\beta}^4\bar{Y}(x) = 0 \tag{3}$$

with

$$\bar{\beta}^4 = \frac{\rho A}{EI}\bar{\Omega}^2 \tag{4}$$

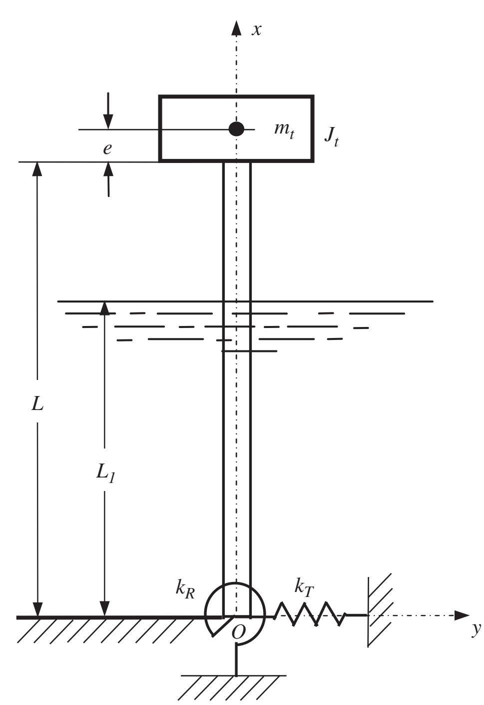

Fig. 1. An immersed beam carrying a tip mass m_t with mass moment of inertia J_t and eccentricity e, and supported by a translational spring k_T and a rotational spring k_R.