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On wave groups in a Gaussian sea

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Abstract

In the context of Gaussian waves, if two successive wave crests of amplitude h_1 and h_2, respectively, are recorded in time at a fixed point $\mathbf{x}_0$ then in the limit of $h_1 \to \infty$ and $h_2 \to \infty$, with probability approaching 1, a wave group has passed closed by the point $\mathbf{x}_0$ at the apex of its development stage, giving rise to an isolated extreme crest. The two large successive wave crests occur at $\mathbf{x}_0$ during the initial phase of decay of the wave group and they are lagged in time by $T_2^* + O(h_1^{-1}, h_2^{-1})$, T_2^* being the abscissa of the second absolute maximum of the time covariance function $\psi(T)$ of the surface displacement.

Thus, either an isolated extreme crest event or two consecutive extreme crest events are particular realizations of the space–time evolution of a wave group, in agreement with the theory of quasi determinism of Boccotti [2000. Wave Mechanics for Ocean Engineering. Elsevier, Oxford].

This result is of relevant interest for offshore engineering. Firstly, the design of offshore structures resisting to a double wave impact can be based on the wave forces generated by the mechanics of a single wave group. On the other hand, in the context of nonlinear water waves, extreme events and their probability of occurrence can be investigated by studying the nonlinear evolution of a wave group.

Keywords: Successive wave crests; Gaussian sea; Quasi-determinism; Wave group; Conditional probability; Extreme event; Nonlinear evolution

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1. Introduction

In the context of Gaussian waves, in the early seventies Lindgren proved that in the time domain, locally to a very high crest, the surface displacement tends to assume the shape of the autocovariance function $\psi(T) = \langle \eta(t)\eta(t+T) \rangle$ where $\langle \cdot \rangle$ is the time average operator (cf. Lindgren, 1970, 1972). Tromans et al. (1991) used this time-domain formulation to analyze wave measurements and renamed it as 'new wave theory'. Wave statistics has been studied by Longuet-Higgins (1952) who proved that the wave heights of a narrow-band Gaussian sea are distributed according to the Rayleigh form. Because of the symmetry of the Gaussian sea state both crest and trough distributions follow the same Rayleigh law for narrow-band spectra. For more general Gaussian processes with finite-band spectra, it is well known that the Rayleigh distribution is an upper bound for the probability of exceedance of both crest heights and crest-to-trough wave heights. It is also well known that the Rayleigh law tends to be asymptotically exact in the limit of large crest amplitudes (cf. Sun, 1993; Maes and Breitung, 1997). As regard to the crest-to-trough wave heights, variants of the Rayleigh distribution which take into account the effects due to the finite bandwidth of the spectrum have been proposed by Longuet-Higgins (1980) and Naess (1985). A rigorous derivation of the exact asymptotic expression for the probability of exceedance of crest-to-trough wave heights, irrespective of bandwidth of the spectrum, have been derived for the first time, in the eighties, by Boccotti (1981, 1982, 1983, 1989) as a corollary of his theory of quasi-determinism. Boccotti (1997, 2000) formulated his theory revealing the mechanics of three-dimensional wave groups and their relation to the occurrence of extreme waves in a Gaussian sea. The theory was verified in the nineties with some small-scale field experiments both for waves in an undisturbed field (cf. Boccotti et al., 1993a) and for waves interacting with structures (cf. Boccotti et al., 1993b). An alternative approach for the derivation of the quasi-determinism theory was proposed by Phillips et al. (1993a) who also obtained a field verification off the US Atlantic coast (Phillips et al., 1993b). There are two versions of the theory of quasi-determinism: the first version deals with the extreme crest height, whereas the second one deals with the extreme wave height. Both the versions are congruent to each other because they both reveal that either an extreme crest height or a wave height are particular realizations of the evolution of a well defined wave group (cf. Boccotti, 2000, 482pp). In particular, an extreme crest occurs at the point $\mathbf{x}_0$ when a wave group passes through $\mathbf{x}_0$ with the crest of its central wave exactly at the envelope center. The wave group has reached its maximal contraction at the point $\mathbf{x}_0$ and after it tends to decay. If an extreme wave height is recorded at the point $\mathbf{x}_0$ instead, it means that the wave group has reached its maximal contraction before the point $\mathbf{x}_0$. In this case, the wave group passes through the point $\mathbf{x}_0$ in its initial phase of decay and the zero downcrossing of the central wave coincides with the envelope center.

As corollary of his theory, Boccotti (1989, 2000) derived the asymptotic form of the probability distribution of the crest-to-trough wave height H_w as

$$\Pr(H_\mathrm{w} > H) = c \exp\left(-\frac{H^2}{4\sigma^2(1+\psi^*)}\right), \quad \frac{H}{\sigma} \to \infty,$$