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A comparative study of Kalman filter and Linear Matrix Inequality based H infinity filter for SPND delay compensation

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ABSTRACT

This paper deals with delay compensation of vanadium Self Powered Neutron Detectors (SPNDs) using Linear Matrix Inequality (LMI) based H-infinity filtering method and compares the results with Kalman filtering method. The entire study is established upon the framework of neutron flux estimation in large core Pressurized Heavy Water Reactor (PHWR) in which delayed SPNDs such as vanadium SPNDs are used as in-core flux monitoring detectors. The use of vanadium SPNDs are limited to 3-D flux mapping despite of providing better Signal to Noise Ratio as compared to other prompt SPNDs, due to their small prompt component in the signal. The use of an appropriate delay compensation technique has been always considered to be an effective strategy to build a prompt and accurate estimate of the neutron flux. We also indicate the noise-response trade-off curve for both the techniques. Since all the delay compensation algorithms always suffer from noise amplification, we propose an efficient adaptive parameter tuning technique for improving performance of the filtering algorithm against noise in the measurement.

1. Introduction

In Pressurised Heavy Water Reactors (PHWRs), the neutron flux distributed over the entire core does not provide an absolute value of reactor power. The neutron flux signal thus, heavily depends on the calibration with respect to some reference power sensed by other signals such as total thermal output of the core. Other factors such as reactivity device fluctuations, non-linear core model, uncertainty about reactivity information and sensor degradation worsen the overall situation. The reactor control or protection systems heavily rely upon the prompt and accurate information of neutronic flux for taking any control or safety action.

The detectors used for sensing the neutron or gamma flux are known as Self Powered Neutron Detectors (SPNDs). These are in-core instruments used for measuring neutron flux inside the reactor core and are primarily used for regulation and protection of the reactor. The neutron interacts with the SPND sensitive material and undergoes several transitions out of which some of the transitions are prompt while others are delayed depending upon the characteristics of the sensitive material. The term "delay" is generally used across the industries to represent the first order lag exhibited by the SPND output. The SPND based on n, β such as vanadium is slow responding and its accuracy in terms of signal to noise ratio is relatively higher as compared to SPNDs based on n, γ, e. The use of vanadium SPNDs has been limited to less critical but important application such as online three dimensional flux mapping along the different geometric planes in the reactor core. Therefore, it is of much interest to develop a robust delay compensation algorithm for n, β based SPNDs to make them useful for safety critical application.

A dynamic delay compensation technique to make use of these SPND signals into a more useful prompt signal has always been considered a good strategy. The most common delay compensation techniques initially used were based on direct inversion of the forward transfer function from input flux to the SPND current output and was introduced by Banda and Nappi (1976) for delay compensation of Rhodium SPND primarily used in Light Water Reactors (LWRs). The direct inversion technique provided a reasonable solution for SPND delay compensation; however, it is a well understood fact that this technique tends to increase the noise in the output signal if the measurement is affected by noise. The later researches in similar line were made to address the above issue by Yusuf and Wehe (1990) and Kulacsy and Lux (1997) by using analog techniques. The widespread development of the Kalman filters introduced by Kalman (1960) and later by Maybeck (1982) and Sorenson (1970), led to development of Kalman filter based delay compensation techniques which performed better than the direct

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inversion technique in terms of noise in the final estimated output. The Kalman filter based solution can be found in Auh (1994) and Kantrowitz (1987). Some recent development in Kalman filter based delay compensation can be found in Srinivasarengan et al. (2012) and Mishra et al. (2013). The Kalman filter has limitation in terms of the knowledge of the noise characteristics and linearity and therefore the robust solutions based on H_∞ filtering was proposed by Park et al. (1999) and the performance was further shown to be improved. However the essential problem of noise amplification has remained unsolved in all the previous research work which obstruct their used for many practical purposes. An adaptive technique based on the fading memory based H_∞ was proposed in Tamboli et al. (2015) which adapts the rate of changes of measured signal unto fading memory forgetting factor. This technique produce much improved results however requires a parallel filter for filtering the noisy measurement and hence requires more computational effort.

The paper deals with two issues in general; first is the formulation for delay compensation along with the trade-off between noise and response time when using the delay compensation algorithms and second issue is the degradation in SNR associated with delay compensation algorithms. For the latter issue we propose the adaptive technique based on the dynamic tuning of the process covariance matrix. The proposed technique in a broad sense controls the filter parameter such that the contribution of delay compensation algorithm exists only when the need arises, reflected in terms of updated process covariance matrix. This way, the noise in the estimated output reduces to a minimum, under the steady state condition (i.e. in the absence of a transient) and the overall signal to noise ratio improves considerably. The proposed technique is less computational intensive as compared to other proposed method. The entire work is done primarily for vanadium SPND however the same can be readily extended to other SPNDs with more complex distribution of delayed fractions.

The paper is structured as follows: in Section 2, we describe the PHWRs in brief followed by the description of SPND and its working principle. In Section 3, we present the detailed derivation and study of the delay compensation methodology followed by simulation studies and the trade-off curves. In Section 4, we present the formulation for the adaptive delay compensation technique based on state difference, followed by the simulation studies. We also present offline simulation experiments on the real time sensor data for demonstrating the performance of the proposed technique.

2. Pressurised Heavy Water Reactor (PHWR) model

The Pressurised Heavy Water Reactor (PHWR) is based on pressure tube concept with heavy water (D_2O) as both coolant and moderator. The core diameter is approximately 7 m and length is approximately 6 m. The detailed description of PHWR is beyond the scope of this paper; however a brief description is necessary before moving on to the flux estimation concepts. The reactor core dynamic behaviour is explained by the Point Kinetic Equations (PKEs) which are derived from the neutron transport equation involving neutron diffusion and generation. The PKEs describe the rate of change of neutron density ($n.cm^{-2}.sec^{-1}$) with respect to reactivity and delayed neutron precursor concentration. The point kinetic model can be written as (Tiwari et al., 1996):

$$\left.\begin{aligned} \frac{dN}{dt} &= \frac{(\rho-\beta_r)}{l}N + \sum_{r=1}^{6}\lambda_r C_r, \\ \frac{dC_r}{dt} &= \frac{\beta_r}{l}C_r - \lambda_r C_r, \end{aligned}\right\} \tag{1}$$

where N is the neutron density (neutrons.cm^{-3}). β_r and λ_r denote the delayed neutron fractional yield and decay constant of the fission product neutron precursor group r. C_r represent the concentration of the precursor group r at time t. The expression for concentration of the precursor group can be replaced with average concentration C replacing β_r and λ_r with their average β and λ. ρ denotes reactivity controlled by the reactivity devices.

The above model explains the behaviour for small core homogeneous model. For large core reactor, the point kinetic model can be expanded to multiple point kinetic equations each controlled by its associated reactivity device and coupled to each other through coupling coefficient as follows:

$$\left.\begin{aligned} \tfrac{dN_i}{dt} &= \tfrac{(\rho_i-\beta)}{l}N_i + \lambda C - \tfrac{1}{l}\textstyle\sum_{j=1}^{n} m_{ij}N_i + \tfrac{1}{l}\textstyle\sum_{j=1}^{n} m_{ji}N_j, \\ \tfrac{dC_i}{dt} &= \tfrac{\beta}{l}N_i - \lambda C, \end{aligned}\right\} \tag{2}$$

where n is the number of cells affected by localized flux variation. Since we have used the normalized flux the N_i is replaced with $N_i/N_F = N_i'$ and C_i will be replaced with $C_i/N_T = C_i'$, where N_T is the flux at full power (i.e. 2×10^{14} nv). The coupling coefficient m_{ij} can be given by:

$$\begin{aligned} m_{ij} &= \frac{D\upsilon l\chi_{ij}}{d_{ij}V_j} \quad (i \neq j), \\ m_{ii} &= 0, \end{aligned} \tag{3}$$

where D is the diffusion coefficient; υ is thermal neutron speed; l is prompt neutron life time; χ_{ij} is the area of the interface between ith and jth zone; d_{ij} the centre to centre distance between ith and jth zone and V_j is volume of the jth zone. The coupling coefficient between a zone and a non-neighbouring zone can be conveniently assumed to be zero. The PKE is a set of stiff non-linear equation since the rate of neutron density variations heavily depends upon the reactivity. The proper zonal reactivity control can be only achieved through accurate and prompt neutron flux information.

2.1. Self Powered Neutron Detectors

The reactor power for large core PHWR cannot be accurately assessed by average flux information measured throughout core neutron flux sensors such as Ionization Chamber. The core condition for such cases is assessed by a number of in-core sensor i.e. self powered neutron detector. Various neutron or gamma sensitive material is used for sensing the in-core flux such as rhodium, cobalt, nickel, platinum or vanadium. The material based on n,γ,e exhibits prompt response whereas material based on n,β such as vanadium and Rhodium exhibits delayed response. The schematic of SPND is shown in the Fig. 1.

Among the above mentioned materials, the SPND with n,β exhibits higher signal strength and hence provides better signal to noise ratio. Due to higher signal strength from vanadium SPND, they provide more accurate readings and are used as a reference to correct less accurate and prompt SPNDs.

The observation model is a first order linear dynamic model with ϕ_k as input and y_k as measured output and can be written as

$$x_k = e^{-Ts/\tau}x_{k-1} + (1-e^{-Ts/\tau})\phi_{k-1} + w_{k-1} \tag{4}$$

$$y_k = (1-k_p)x_k + k_p\phi_k, \tag{5}$$

where τ = 325 s for vanadium SPND and k_p is the prompt component fraction which is equal to 0.07. The above expression is normalised and brings the state and input vector to same dimensions and is useful for building the adaptive technique later discussed in the paper. The model can be generalised for the signal containing various components with different time constant $\{\tau_1,\tau_2\ldots\}$ and different proportion $\{k_1,k_2\ldots\}$. Fig. 2 shows the typical histogram for 50,000 data points of noise amplitude experimentally obtained for