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Analytic function expansion nodal (AFEN) method for solving multigroup neutron simplified P_3 (SP_3) equations in hexagonal-z geometry

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ABSTRACT

In this paper, analytic function expansion nodal (AFEN) method was developed to solve multi-group and multi-dimensional neutron simplified P_3 (SP_3) equation in reactor cores with hexagonal fuel assembly. In this method, the intranodal fluxes are expanded into a set of analytic basis functions for each group and Legendre moment. Fourteen boundary conditions has been considered that constrain the intranodal flux distributions in the hexagonal-z node, which include twelve radial surface-averaged partial currents and two axial surface-averaged partial currents. The code takes few-groups cross sections produced by a lattice code and calculates the effective multiplication factor (k_{eff}), flux in multi-group energy, reactivity, and the relative power density at each fuel assembly. Finally, the solution accuracy is tested for the two and three dimensional IAEA benchmark problem. The 3D IAEA problem has been calculated for three different cases. The numerical results demonstrate that the SP_3 AFEN method exhibits better accuracy compared to diffusion method especially when the control materials are inserted in the core.

1. Introduction

The solution of linear transport problems is difficult because of the rich phase space; in general there are seven independent variables: three for space, two describing a direction on the unit sphere, one for speed or energy, and one for time (McClarren, 2011). Obtaining energy and time dependent solutions to 3D transport problems is still challenging, even on petascale computers (McClarren, 2011). The simplified P_N (SP_N) method was developed by Gelbard (1962) when computer resources were used to solve 3D diffusion problems. The SP_N method is a simple extension of the one-dimensional P_N equations to the multidimensional P_N equations. Larsen et al. (1993, 1996) showed that the SP_N approximation is an asymptotic correction to standard diffusion theory. This method greatly reduced complexity of the multidimensional spherical harmonics (P_N) method.

The simplified P_3 (SP_3) approximation to the multigroup neutron transport equation in arbitrary geometries was derived by (Brantley and Larsen, 2000) using a variational analysis. They showed that the SP_3 approximation can eliminate much of the inaccuracy in the diffusion and SP_2 calculations of mixed-oxide (MOX) fuel problems (Brantley and Larsen, 2000). Ryu and Joo (2013) solved SP_3 equations by finite element method for general geometry applications. Their calculation results demonstrated that the SP_3 formulation is particularly beneficial for fast neutron systems.

Because the SP_3 equations have higher accuracy than diffusion theory and also less computational expense than the solution of transport equation with the discrete ordinate (S_N) or spherical harmonics (P_N) approximations, they were utilized in several pin-by-pin whole core calculation codes such as PARCS (Downar et al., 2004, 2009), SCOPE2 (Sugimura et al., 2006), DYN3D (Beckert and Grundmann, 2008) and COCAGNE (Fliscounakis et al., 2012).

For some calculations such as fuel burnup, fuel management or transient analysis, due to the need for many full core spatial solutions, direct few group finite difference solution is inefficient or even impractical (Stacey, 2007). For this reason the nodal solution methods for the SP_3 equations were developed (Downar et al., 2004, 2009; Beckert and Grundmann, 2008; Kim et al., 2009; Cho and Cho, 2010, Jalili et al., 2015a).

In development of the PARCS (Downar et al., 2004, 2009) code, the nodal expansion method (NEM) was utilized for the multigroup SP_3 equation. In this method, the three-dimensional SP_3 equations are changed to three type one-dimensional SP_3 equations by transverse integration procedure. The one-dimensional flux moments were approximated in 4th-order polynomials and

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the transverse leakage shape was approximated in a second-order polynomial for both 0th and 2nd flux moments.

In development of the DYN3D (Beckert and Grundmann, 2008) code, the SP_3 equations are reduced to two-dimensional equations in radial plane (x, y), and one-dimensional equations in axial z-direction by transverse integration over the node height and rectangular node area, respectively. Polynomial and exponential functions were used as intranodal flux expansions.

In the work of (Kim et al., 2009), the conformal mapped nodal simplified P_3 equations were derived and implemented for the two-dimensional neutronics analysis of fast reactor cores with hexagonal fuel assemblies.

In the past twenty years, the analytic function expansion nodal (AFEN) method (Cho and Noh, 1994, 1995; Noh and Cho, 1996; Cho et al., 1997; Woo et al., 2001; Xia and Xie, 2006; Cho and Lee, 2006; Cho and Cho 2010; Jalili et al., 2015a,b) has been the most commonly used for analysis of neutronics core calculations. This method represents multidimensional intra nodal flux distribution in terms of analytic basis functions at any points in the node.

In the work of Cho and Cho (2010), the AFEN method has been reformulated for SP_3 equations and implemented in a version of the AFEN method code COREDAX for cubic geometry (Cho and Cho, 2010).

In the work of Jalili et al. (2015a,b), the new AFEN method has been developed for SP_3 equations in rectangular-z geometry. In this work, the nodes are coupled through the surface averaged partial currents at each nodal interface, in other words, six boundary conditions at each group and Legendre moments have been considered.

In the present work, a multigroup AFEN code named MGHANSP3 is developed for solution of SP_3 equations in hexagonal-z fuel lattice. The boundary conditions include twelve radial surface averaged partial currents across the half radial hexagonal surfaces and two axial surface averaged partial currents. Finally, the solution accuracy is examined for the two and three dimensional IAEA (Herbert, 2008) problem. The numerical results illustrated that the solution of SP_3 equations by the AFEN method is an accurate method for predicting multiplication factor and power distribution in reactor core with hexagonal fuel assembly.

2. Description of the method

2.1. Intranodal flux expansion

The governing equations of the multidimensional SP_3 equation is as follows (Brantley and Larsen, 2000):

$$\begin{aligned}
&\nabla.\phi_1^g + \Sigma_{rg}\phi_0^g = S_0^g,\\
&\tfrac{2}{3}\nabla\phi_2^g + \tfrac{1}{3}\nabla\phi_0^g + \Sigma_{trg}\phi_1^g = 0,\\
&\tfrac{3}{5}\nabla.\phi_3^g + \tfrac{2}{5}\nabla\cdot\phi_1^g + \Sigma_{tg}\phi_2^g = 0,\\
&\tfrac{3}{7}\nabla\phi_2^g + \Sigma_{tg}\phi_3^g = 0,\\
&S_0^g = \sum_{g'}\Sigma_{sg'}\phi_0^g + \tfrac{\chi_g}{k_{eff}}\sum_{g'}\nu\Sigma_{fg'}\phi_0^{g'}
\end{aligned} \tag{1}$$

The zeroth and second Legendre moments of the angular flux in group g (ϕ_0^g, ϕ_2^g) are the scaler variables, and the first and third Legendre moments of the angular flux in group g (ϕ_1^g, ϕ_3^g) are the vector variables. The Eq. (1) in each group can be rewritten as follows (Downar et al., 2009):

$$\begin{aligned}
&\begin{bmatrix} -D_1\nabla^2 + \Sigma_r & -2\Sigma_r \\ -\tfrac{2}{3}\Sigma_r & -D_3\nabla^2 + \Sigma_{rt} \end{bmatrix}\begin{bmatrix}\Phi_0\\ \phi_2\end{bmatrix} = S_0\begin{bmatrix}1\\ -\tfrac{2}{3}\end{bmatrix},\\
&\Phi_0 = \phi_0 + 2\phi_2, \Sigma_{rt} = \tfrac{5}{3}\Sigma_t + \tfrac{4}{3}\Sigma_r, J_1 = -D_1\nabla\Phi_0, J_3 = -D_3\nabla\phi_2
\end{aligned} \tag{2}$$

$$\Sigma_r, J_1 = -D_1\nabla\Phi_0, J_3 = -D_3\nabla\phi_2$$

The relationship between partial currents, surface fluxes and net currents is as follows (Downar et al., 2009):

$$j_1^{\pm} = \frac{1}{4}\Phi_{0s} \pm \frac{1}{2}J_1 - \frac{3}{16}\phi_{2s}, \quad j_3^{\pm} = \frac{7}{16}\phi_{0s} \pm \frac{1}{2}J_3 - \frac{1}{16}\Phi_{0s} \tag{3}$$

Eq. (2) can be rewritten in the form of matrix equation as follows (Cho and Cho, 2011):

$$-\nabla\cdot[D]\nabla\bar{\phi}(\bar{r}) + [A]\bar{\phi}(\bar{r}) - \frac{\bar{\chi}}{k_{eff}}[F^T]\bar{\phi}(\bar{r}) = 0 \tag{4}$$

where

$$\begin{aligned}
&\bar{\phi}(\bar{r}) = \{[\phi_1], [\phi_2], \ldots, [\phi_G]\}^T\\
&[D] = \begin{bmatrix} [D_1] & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & [D_G] \end{bmatrix}\\
&[A] = \begin{bmatrix} [A_{11}] & [A_{12}] & \cdots & [A_{1G}] \\ [A_{21}] & \ddots & & \vdots \\ \vdots & & \ddots & \vdots \\ [A_{G1}] & [A_{G2}] & & [A_{GG}] \end{bmatrix}\\
&[F]^T = [[F_1]^T[F_2]^T \ldots [F_G]^T]
\end{aligned}$$

and

$$\begin{aligned}
&[\phi_g] = [\Phi_0^g, \phi_2^g]\\
&[D_g] = \begin{bmatrix} D_0^g & 0 \\ 0 & D_2^g \end{bmatrix}\\
&[A_{gg'}] = \begin{bmatrix} -\Sigma_s^{g'\to g} & 2\Sigma_s^{g'\to g} \\ \tfrac{2}{3}\Sigma_s^{g'\to g} & -\tfrac{4}{3}\Sigma_s^{g'\to g} \end{bmatrix}\\
&[A_{gg}] = \begin{bmatrix} \Sigma_r^g & -2\Sigma_r^g \\ -\tfrac{2}{3}\Sigma_r^g & \tfrac{5}{3}\Sigma_t^g + \tfrac{4}{3}\Sigma_r^g \end{bmatrix}\\
&[F_g]^T = \begin{bmatrix} \nu\Sigma_f^g & 2\nu\Sigma_f^g \end{bmatrix}
\end{aligned} \tag{5}$$

We can rewrite Eq. (4) in the form of the following equation:

$$-\nabla^2\bar{\phi}(\bar{r}) + [A']\bar{\phi}(\bar{r}) = 0 \tag{6}$$

where

$$[A'] = [D]^{-1}\left([A] - \frac{1}{k_{eff}}\bar{\chi}[F]^T\right) \tag{7}$$

For analytic solution of Eq. (1), the eigenvalues λm and thus eigenvectors u_m of the matrix [A′] must be calculated. For general multigroup problems the eigenvalues (and thus eigenvectors) may be real or complex.

In this project, the QR algorithm is used to find the eigenvalue evaluation. The QR decomposition is based on Householder transformations.

If all eigenvalues λm of the matrix [A′] are real, to reduce Eq. (6) to the following decoupled form:

$$\nabla^2\Psi_m - \lambda_m\Psi_m = 0, \quad m = 1, 2, \ldots, 2\times G \tag{8}$$

we can use the following transformation

$$\begin{aligned}
&U = [u_1, \ldots, u_m, \ldots, u_{2G}], u_m = [u_m^1, \ldots, u_m^{2G}]^T\\
&\Psi(r) = U^{-1}\Phi(r)
\end{aligned} \tag{9}$$

where $\Psi(r)$ denote auxiliary flux.