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The energy dependent Pál–Bell equation and the influence of the neutron energy on the survival probability in a supercritical medium

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ABSTRACT

The survival probability of a neutron injected into a supercritical fissile medium is studied with respect to the energy dependence of the incoming neutron. We assume a point model but allow the energy dependence to be included through a general energy exchange model. We have studied the effect of slowing down on the survival probability by means of the Pál–Bell equation and the Goertzel-Greuling kernel, the separable kernel, the Hansen–Roach dataset and a WIMS generated dataset for a homogenised reactor to approximate the slowing down process. The Goertzel-Greuling model is known to be exact for $A = 1$ and reverts to age theory for large mass ratios. It is also accurate for all intermediate mass numbers, except possibly when strong resonances are present. The separable kernel is a simple model of energy exchange, corresponding to the thermalisation of neutrons in a single collision, which ignores the slowing down process but provides a simple result allowing both ends of the reactor spectrum to be included. The Hansen–Roach data set is obtained from a realistic slowing down model in a fast system and the thermal system is modelled by homogenised reactor data, generated using WIMS, and is typical of the material found in a PWR. Using a scattering cross section which is an arbitrary function of energy, and capture and fission cross sections which are proportional to $1/v$, we find that the survival probability is energy independent insofar as it depends only on the values of the ratio of the fission and capture cross sections. For non-$1/v$ cross sections there is an energy dependence which we discuss below. The formalism developed is robust enough for studies to be made of the influence of resonance cross sections and inelastic scattering on survival probability.

1. Introduction

The survival probability of a neutron injected into a supercritical fissile medium has long been a subject of study in connection with low source start up (Hurwitz et al., 1963; Bell, 1963) and fast burst reactors such as Godiva (Wimett et al., 1960; Hansen, 1960). A complete theory of this subject has been developed over the years, in particular by Bell (1965) and Pál (1958, 1962) and is summarised comprehensively in Pázsit and Pál (2008). The general theory encompasses a detailed energy, space and time dependence of the probability function but, in many practical cases, it is adequate to simplify this to the point model which is independent of space and energy. The point model assumes a global average over space, angle and energy and it is of some interest to examine the influence of the energy dependence of the cross sections and the slowing down process on the results obtained from it. These might be expected to be large as neutrons are born in fission in the MeV energy region and end up, after slowing down, in the thermal region ∽0.025 eV; thus a very large energy span is involved. We will study this matter by considering the survival probability of a neutron injected into an infinite medium at a given speed and examine the influence of the slowing down process on the outcome. For simplicity, we assume in one case that the material is pure ^{235}U and in the other case that it is a homogeneous mixture of material typical of a PWR.

We will show that if the absorption and fission cross sections follow the $1/v$ law, then the survival probability is independent of the slowing down model. For more realistic variations of cross section energy dependence, the survival probability is found to be a function of the slowing down model. Some explicit examples of this behaviour are given by developing a numerical method using three different models for the scattering process: Goertzel-Greuling, separable kernel (Williams, 1966) and finally the use of explicit scattering cross sections from both the Hansen and Roach dataset (1961) for a fast system and a WIMS generated dataset for a thermal system.

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2. General theory

2.1. Energy dependent Pál–Bell equation and its influence on the point model results

The complete Pál–Bell equation, as derived by Pázsit and Pál (2008), for the generating function

$$G(z,t,R|\vec{r}_0,\vec{v}_0,s)=\sum_{n=0}^{\infty}z^nP(n,t,R|\vec{r}_0,\vec{v}_0,s) \tag{1}$$

is given by

$$\frac{\partial G(z,t,R|\vec{r}_0,\vec{v}_0,s)}{\partial s}+\widehat{T}G(z,t,R|\vec{r}_0,\vec{v}_0,s)$$
$$+\lambda_f(\vec{r}_0,\vec{v}_0,s)f\left[G_p(z,t,R|\vec{r}_0,s),\vec{v}_0\right]+\lambda_c(\vec{r}_0,\vec{v}_0,s)=0 \tag{2}$$

where $\lambda_a=\lambda_c+\lambda_f$

$$\widehat{T}G(z,t,R|\vec{r}_0,\vec{v}_0,s)=-(\lambda_a(\vec{r}_0,\vec{v}_0,s)+\lambda_s(\vec{r}_0,\vec{v}_0,s))G(z,t,R|\vec{r}_0,\vec{v}_0,s)$$
$$+\vec{v}_0.\nabla_{\vec{r}_0}G(z,t,R|\vec{r}_0,\vec{v}_0,s)+\lambda_s(\vec{r}_0,\vec{v}_0,s)$$
$$\times\int d\vec{v}'g(\vec{v}_0\to\vec{v}')G(z,t,R|\vec{r}_0,\vec{v}',s) \tag{3}$$

where $g(\vec{v}_0\to\vec{v}')$ is the slowing down kernel and

$$G_p(z,t,R|\vec{r}_0,s)=\int d\vec{v}_0{}'F_0(\vec{v}_0)G(z,t,R|\vec{r}_0,\vec{v}_0{}',s) \tag{4}$$

$f(x,\vec{v}_0)$ defines the random emission of neutrons in a fission event and is defined by

$$f(x,\vec{v}_0)=\sum_{k=0}^{K}\frac{(-1)^k}{k!}\chi_k(\vec{v}_0)(1-x)^k \tag{5}$$

with the multiplicity χ_k, given in terms of the probability of v neutrons emitted in a fission event p_v, by

$$\chi_k=\sum_{v=k}^{K}\frac{v!}{(v-k)!}p_v \tag{6}$$

Note that we use slightly different notation to Pázsit and Pál in that they use Q to denote the transition rate and we use λ. Also, Pázsit and Pál use Q_a for their capture rate while we use λ_c, with $\lambda_a=\lambda_c+\lambda_f$. $F_0(\vec{v})$ is the fission spectrum of the prompt neutrons. The final conditions associated with Eq. (2) are

$$G(z,t,R|\vec{r}_0,\vec{v}_0,t)=1-(1-z)\Delta(\vec{r}_0,V_r)\Delta(\vec{v}_0,U_v) \tag{7}$$

where

$$\Delta(\vec{u}_0,U)=1\quad\text{if}\quad\vec{u}_0\in U\quad\text{and}\quad\Delta(\vec{u}_0,U)=0\quad\text{if}\quad\vec{u}_0\notin U \tag{8}$$

$G(z,t,R|\vec{r}_0,\vec{v}_0,s)$, as defined above is the 'single particle generating function' and is always time dependent as it relates to the chain initiated by a single neutron. V_r is a sub-region within the reactor and U_v is an energy range; generally the complete range. The usefulness of G is that it does not change when different sources are used and, in a loose sense, is analogous to a Greens function. To relate G to the case where there is an independent source, we proceed as follows. Such a source can itself emit varying numbers of neutrons at each disintegration and this will also be dealt with in the same way as for the forward equation. Now if the source emits neutrons with a compound Poisson distribution with varying multiplicity, we may write for the source generating function (Bartlett, 1955)

$$G_S(z,t,R|s)=exp\left[\int_s^t ds'\int_{V_{\vec{r}}}d\vec{r}\int_{U_{\vec{v}}}d\vec{v}S_d(\vec{r},\vec{v},s')\left[f_q(G(z,t,R|\vec{r},\vec{v},s'))-1\right]\right]$$
$$=\sum_{N=0}^{\infty}z^Np_s(N,t|s) \tag{9}$$

with $f_q(z)$ the generating function for the source emission and S_d the disintegration rate of the process that leads to the source neutrons. Note that while G is always time dependent G_S with be asymptotically time-independent for a sub-critical system. The equation for G_S can also be written

$$-\frac{\partial G_S(z,t,R|s)}{\partial s}=\int_{V_{\vec{r}}}d\vec{r}\int_{U_{\vec{v}}}d\vec{v}S_d(\vec{r},\vec{v},s)\left[f_q(G(z,t,R|\vec{r},\vec{v},s))-1\right]G_S(z,t,R|s) \tag{10}$$

We will not be concerned with an independent source but cite the above generalisation for completeness.

Although the Pál–Bell equation is given in terms of $G(z,t,R|\vec{r}_0,\vec{v}_0,s)$, it is usually more convenient in practice to use the modified generating function $\widetilde{G}(z,t,R|\vec{r}_0,\vec{v}_0,s)=1-G(z,t,R|\vec{r}_0,\vec{v}_0,s)$. The reason for this is evident from Eq. (5), which is a rapidly converging series for many practical situations. Indeed, taking only terms up to $k=2$, leads to the well-known and accurate quadratic approximation.

For the problem under consideration, we are only interested in the infinite medium time dependent case. In this instance we may neglect the spatial operator and consider an isotropic distribution of neutrons. We also assume that the cross sections themselves are time independent which leads to

$$\widetilde{G}(z,t|v,s)\to\widetilde{G}(z,v,t) \tag{11}$$

$$\left[\frac{1}{v}\frac{\partial}{\partial t}+\Sigma(v)\right]\widetilde{G}(z,v,t)=\Sigma_s(v)\int_0^{\infty}dv'g(v\to v')\widetilde{G}(z,v',t)$$
$$-\Sigma_f(v)\sum_{k=1}^{K}\frac{(-1)^k}{k!}\chi_k\left[\int_0^{\infty}dv'F(v')\widetilde{G}(z,v',t)\right]^k \tag{12}$$

where $\Sigma=\Sigma_a+\Sigma_s,\Sigma_a=\Sigma_c+\Sigma_f$ and $F(v)$ is the fission spectrum. The initial condition is $\widetilde{G}(z,v,0)=1-z$. It is convenient to convert this equation to the energy variable, where we have

$$\left(\frac{1}{v}\frac{\partial}{\partial t}+\Sigma(E)\right)\widetilde{G}(z,E,t)=\Sigma_s(E)\int_0^{\infty}dE'g(E\to E')\widetilde{G}(z,E',t)$$
$$+\Sigma_f(E)f(\zeta(z,t)) \tag{13}$$

with

$$f(\zeta)=-\sum_{k=1}^{K}\frac{(-1)^k}{k!}\chi_k\zeta^k\qquad\zeta(z,t)=\int_0^{\infty}dE'F(E')\widetilde{G}(z,E',t) \tag{14}$$

Note that the linear part of Eqs. (12) and (13) is the adjoint of the normal slowing down equation.

Let us multiply by v and re-write Eq. (13) as

$$\left(\frac{\partial}{\partial t}+v\Sigma(E)\right)\widetilde{G}(z,E,t)=v\Sigma_s(E)\int_0^{\infty}dE'g(E\to E')\widetilde{G}(z,E',t)$$
$$+v\Sigma_f(E)f(\zeta(z,t)) \tag{15}$$

Suppose the absorption and fission cross sections are proportional to $1/v$ so that we have $v\Sigma_a(E)=\lambda_a, v\Sigma_f(E)=\lambda_f$ and $v\Sigma_s(E)=\lambda_s(E)$ where $\lambda_s(E)$ is an arbitrary function of E. Then Eq. (15) becomes

$$\left(\frac{\partial}{\partial t}+\lambda_a+\lambda_s(E)\right)\widetilde{G}(z,E,t)=\lambda_s(E)\int_0^{\infty}dE'g(E\to E')\widetilde{G}(z,E',t)$$
$$+\lambda_f f(\zeta(z,t)) \tag{16}$$

Suppose we now assume that $\widetilde{G}(z,E,t)=\theta(z,t)$ and insert this into Eq. (16). We then find