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Nonoverlapping local/global iterations with 2-D/1-D fusion transport kernel and p-CMFD wrapper for transient reactor analysis—II: Parallelization and Predictor–Corrector Quasi-Static method application

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ABSTRACT

As modern computing power grows, whole-core transport calculations become more viable with parallel computing architectures. Nonoverlapping local/global (NLG) iterative method has been recently developed for the whole-core transport calculation. The NLG iteration adopts the 2-D/1-D fusion transport kernel as the local solver, and the p-CMFD equation is used as the global wrapper. The NLG iteration is capable of solving 3-D transient heterogeneous problems, and it is naturally parallelizable. In this study, the computationally-expensive local problems in the NLG iteration are parallelized by MPI protocol, and the Predictor–Corrector Quasi-Static (PCQS) method is applied to the transient calculations to reduce the computing time further. The parallelized NLG iteration and the PCQS method have been implemented in an in-house code, CRX-2K. Several numerical problems are computed, and the numerical results reveal that the parallelized NLG iteration has high potential in the realistic whole-core transport calculation, and the computing time in the transient calculations can be reduced by the PCQS method if the reactivity behaves linearly in time for a given macro time-step size.

1. Introduction

Multi-group diffusion nodal methods with assembly-wise nodal parameters have been used as a tool for conventional reactor core analysis. The nodal parameters are obtained by isolated single-assembly transport calculations under all-reflective boundary condition, so the inter-assembly transport effects are not well reflected. As computing power grows both in CPU clock speed and computing memory size, the direct whole-core transport calculation becomes attractive.

Despite the computing power growth, the direct whole-core transport calculation is not practical yet due to its tremendous computational burden (Smith et al., 2003; Hoogenboom et al., 2010) with current computing systems. However, the whole-core transport calculation could be viable if a proper method is powered by modern parallel hardware architectures. Note that the computing power growth is physically limited by CPU clock speed, but it is growing by the adoption of parallel hardware architectures (Sutter, 2005).

As an effort to use the modern parallel hardware architecture in the whole-core transport calculation, the nonoverlapping local/global (NLG) iterative method (in short, the NLG iteration) has been developed (Yuk et al., 2013; Yuk and Cho, 2014, 2015), and extended to transient calculations recently in a companion paper (Cho and Cho, 2015). The NLG iteration is naturally parallelizable, so the implementation of parallel computing is straightforward. In the NLG iteration, the local problems are solved by the 2-D/1-D fusion transport method (Cho et al., 2002, 2003; Lee and Cho, 2006; Lee, 2006), and the p-CMFD method (Cho et al., 2003; Cho, 2012) is used as the global wrapper. The solution by the NLG iteration is 3-D transport solution in fine-mesh level, since the 2-D/1-D fusion kernel does not need any homogenization in a cell level.

In the transport transient calculations, the time-step size should be in millisecond-order to obtain accurate solutions (Cho and Cho, 2015; Cho et al., 2005; Rineiski and Doriath, 1997; Zhu et al., 2015a), so the transient calculations usually consume much longer computing time than the steady-state calculation (Cho and Cho, 2015). Therefore, a larger time-step size is inevitable to reduce the computing time, but a larger time-step size will obviously lead to an inaccurate solution. The Predictor–Corrector Quasi-Static (PCQS) method (Dulla et al., 2008) may preserve the solution

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accuracy with a larger time-step size, since the PCQS method factorizes the angular flux into a slow varying (in time) "shape function" and an "amplitude function".

In this paper, the local problems (i.e., the 2-D/1-D fusion transport kernel) in the NLG iteration are parallelized by Message Passing Interface (MPI) protocol (Gabriel et al., 2004). Therefore, the heavy computational burden loaded in the local transport calculations can be distributed over parallel computing nodes, and consequentially the total computing time will be reduced. In addition, the PCQS method is applied to the NLG iteration to reduce the computing time further in the transient calculations, and the adjoint angular flux required in the PCQS method to formulate the point kinetics (PK) equations (Dulla et al., 2008; Caron et al., 2016) is approximated by the solution of a steady-state adjoint p-CMFD equation. As a sequel to the companion paper (Cho and Cho, 2015), this paper is organized as follows. In Section 2, the parallelization of the NLG iteration is described. In Section 3, the PCQS method for the NLG iteration is derived. Numerical results are shown in Section 4, and conclusions are presented in Section 5.

2. Parallelized nonoverlapping local/global iterative method

In the transient NLG iteration, the local problems are solved by the transient 2-D/1-D fusion transport kernel (abbreviated by the 2D1D FTK, here and after), and the local problems are wrapped up by the transient p-CMFD equation as the global wrapper. In this section, the formulation of the 2D1D FTK and the corresponding p-CMFD equation are derived. The derivational procedures are extensively covered by the companion paper (Cho and Cho, 2015), so some derivational procedures are simplified. The way of parallelization and the proper local problem size are also discussed in this section.

2.1. Local problem: transient 2-D/1-D fusion transport kernel

Let us consider the following time-dependent neutron transport equation with delayed neutron precursors:

$$\frac{1}{v_g}\frac{\partial\psi_g(\vec{r},\vec{\Omega},t)}{\partial t}+\vec{\Omega}\cdot\nabla\psi_g(\vec{r},\vec{\Omega},t)+\sigma_{t,g}\psi_g(\vec{r},\vec{\Omega},t)$$
$$=\frac{1}{4\pi}\left[\frac{(1-\beta)}{k_{eff}}\chi_{p,g}\sum_{g'}\nu\sigma_{f,g'}\phi_{g'}(\vec{r},t)+\sum_{g'}\sigma_{s0,g'\to g}\phi_{g'}(\vec{r},t)\right.$$
$$\left.+\sum_d\chi_{d,g}\lambda_d C_d(\vec{r},t)\right], \tag{1}$$

and

$$\frac{\partial C_d(\vec{r},t)}{\partial t}=-\lambda_d C_d(\vec{r},t)+\frac{\beta_d}{k_{eff}}\sum_{g'}\nu\sigma_{f,g'}\phi_{g'}(\vec{r},t), \tag{2}$$

with the following boundary condition and initial condition:

B.C.: $\psi_g(\vec{r},\vec{\Omega},t)$ is given for $\vec{r}\in\partial D$, $\vec{\Omega}\cdot\vec{n}_s<0$,

I.C.: $\psi_g(\vec{r},\vec{\Omega},0)\equiv\psi_{g,0}(\vec{r},\vec{\Omega})$, which is obtained by the steady-state calculation,

where g is the group index, d is the delayed neutron precursor family index, C_d is the delayed neutron precursor concentration of family d, $\vec{n}_s$ is outgoing normal vector subjected to surface s, and the other notations are standard.

To derive angle- and time-discretized neutron transport equation at the time step t_{n+1}, several approximations are introduced: (*i*) The time derivative term in Eq. (1) is approximated by the fully implicit method, and the angular flux in the derivative term is assumed to be isotropic to avoid huge memory requirement (Cho et al., 2005; Talamo, 2013), (*ii*) the fission source term in Eq. (2) is approximated by a second-order polynomial in time, and is integrated analytically to eliminate the delayed neutron precursor concentration unknown (Joo et al., 1998), and (*iii*) the continuous angle domain is discretized by the product quadrature (Lewis and Miller, 1984) in the azimuthal angle and the TY quadrature (Yamamoto et al., 2007) in the polar angle. With the approximations, the following angle- and time-discretized neutron transport equation at the time step t_{n+1} with the angle direction $\vec{\Omega}_j$ is written as follows:

$$\sin\theta_j\frac{\partial\psi_{g,j}(\vec{r})}{\partial p}+\xi_j\frac{\partial\psi_{g,j}(\vec{r})}{\partial z}+\sigma_{t,g}\psi_{g,j}(\vec{r})=q_g(\vec{r}), \tag{3}$$

where

$$q_g(\vec{r})=(\alpha_g+(1-\beta)\chi_{p,g})F(\vec{r},t_{n+1})+\sum_{g'}\sigma_{s0,g'\to g}\phi_{g'}(\vec{r},t_{n+1})+S_g(\vec{r},t_n)$$
$$-\frac{1}{v_g}\frac{\phi_g(\vec{r},t_{n+1})-\phi_g(\vec{r},t_n)}{\Delta t_{n+1}},$$

$$\alpha_g=\sum_d\chi_{d,g}\beta_d\varsigma_{d,n+1},$$

$$S_g(\vec{r},t_n)=\sum_d\chi_{d,g}\tau_d\lambda_d C_d(\vec{r},t_n)+\sum_{l=n-1}^{n}\sum_d\chi_{d,g}\beta_d\varsigma_{d,l}F(\vec{r},t_l),$$

$$F(\vec{r},t_l)=\frac{1}{k_{eff}}\sum_{g'}\nu\sigma_{f,g'}\phi_{g'}(\vec{r},t_l),$$

$$\varsigma_{d,n-1}=\frac{1}{\lambda_d\Delta t_n(\gamma+1)}\left[\frac{2\tilde{\tau}_d}{\lambda_d\Delta t_n}-\gamma(\tau_d+1)\right],$$

$$\varsigma_{d,n}=\frac{1}{\lambda_d\Delta t_n}\left[\tau_d+1+\frac{\tilde{\tau}_d}{\gamma}\left(1-\frac{2}{\lambda_d\Delta t_n}\right)\right]-\tau_d,$$

$$\varsigma_{d,n+1}=1-\frac{2}{\lambda_d\Delta t_n(\gamma+1)}+\frac{\tilde{\tau}_d}{\lambda_d\Delta t_n(\gamma+1)\gamma}\left(\frac{2}{\lambda_d\Delta t_n}-1\right),$$

and $C_d(\vec{r},t_n)$ is the delayed neutron precursor concentration at time step t_n, and other quantities are defined in the companion paper (Cho and Cho, 2015). The time step index is omitted for the sake of brevity. Eq. (3) can be rewritten in the radial and the axial equations as follows without any approximation:

$$\sin\theta_j\frac{\partial\psi_{g,j}(\vec{r})}{\partial p}+\sigma_{t,g}\psi_{g,j}(\vec{r})=q_g(\vec{r})-\xi_j\frac{\partial\psi_{g,j}(\vec{r})}{\partial z}, \tag{4}$$

and

$$\xi_j\frac{\partial\psi_{g,j}(\vec{r})}{\partial z}+\sigma_{t,g}\psi_{g,j}(\vec{r})=q_g(\vec{r})-\sin\theta_j\frac{\partial\psi_{g,j}(\vec{r})}{\partial p}. \tag{5}$$

To solve the complementary equations numerically, the space domain is discretized into radial flat source regions (FSRs) and axial 2-D homogeneous planes. In a FSR, the source terms in Eqs. (4) and (5) are given to be constant in the radial direction (yet angle-dependent). After the right-hand sides of Eqs. (4) and (5) are integrated and averaged in (*i*) the axial direction over the 2-D homogeneous k-th plane and (*ii*) the radial direction over the FSR m respectively, the following "consistently directional decomposed and integrated" 2-D and 1-D equations in the FSR m in the k-th plane are obtained:

$$\sin\theta_j\frac{\partial\psi_{g,j,k}(p)}{\partial p}+\sigma_{t,g,k,m}\psi_{g,j,k}(p)=q_{g,k,m}-L^z_{g,j,k,m}, \tag{6}$$