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Efficient computational system for transient neutron diffusion model via finite difference and theta methods

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ABSTRACT

The space–time neutron diffusion equations with two energy groups and average one group of delayed neutrons are a system of stiff partial differential equations. The efficient computational system is presented to solve the neutron diffusion equations based on finite difference and theta methods. Finite difference method is used to reduce the partial differential equations to the ordinary differential equations. These ordinary differential equations are rewritten in a matrix form. Theta method is developed using the eigenvalues and corresponding eigenvectors of the coefficient matrix. These eigenvalues and the corresponding eigenvectors are calculated analytically. The efficient computational system is applied to multi-dimensional transient neutron diffusion equations with two energy groups and one group of delayed neutrons in the homogenous and heterogenous nuclear reactors. The results of the proposed method are in agreement with the results of traditional methods. The efficient computational system reduces the computational time (CPU) by about 30% compared with the faster reference method. So, the efficient computational system is considered a fast technique more than theta method and traditional numerical codes.

1. Introduction

For nuclear reactors safety, scientists are looking for an accurate knowledge of the time-dependent spatial neutron flux density in nuclear reactors. For this reason, accurate and efficient numerical codes are required for solving the space–time neutron diffusion equations in nuclear reactors. In nuclear reactor problems, especially those involving safety considerations, the coefficients of the neutron diffusion equations depend on parameters such as the neutron power level, precursor concentration of delayed neutrons groups, time, space and other parameters. The space–time neutron diffusion equations with two energy groups and average one group of delayed neutrons consider a stiff system of the partial differential equations. Search for the efficient and accurate methods is considered complicated. Then, the aim is to develop efficient and accurate technique to solve this stiff system.

To substantiate the feasibility and efficiency of the proposed method, results should be compared with results of the traditional codes. The TWIGL code (Yasinsky et al., 1968; Hageman and Yasinsky, 1969) is one of important traditional codes. The TWIGL program uses a specified θ-method of time-differencing with the fluxes at each time step being determined by the cyclic Chebyshev polynomial iterative method. The LUMAC code (McCormick, 1969) assumes that the variation in the neutron flux at each mesh point can be represented as an exponential function of time over each integration time step. Additionally, the assumption is made that the transverse leakage in one spatial direction can be approximated by a pointwise transverse buckling over one time step. The MITKIN method (Reed and Hansen, 1970) uses a particular alternating direction semi-implicit method referred to as an alternating direction explicit method. It has incorporated an exponential transformation of variables which improves the truncation error. The SADI code (Wight et al., 1971) is a matrix splitting method based on an alternating direction implicit scheme. The 3DKIN code (Ferguson and Hansen, 1973) is the nonsymmetric alternating direction explicit method which is extended to three dimensional geometries. The MADI method (Chen et al., 1992) is a mixed implicit–explicit three level alternating direction implicit method. The temporal subdomain method TSM (Utku and Christenson, 1994) is based on a spatial finite element formulation. Padé11 approximation method (Aboanber and Nahla, 2006) applies the Crank–Nicholson approximation for the exponential function of the coefficients matrix for all mesh points of the reactor. Adaptive matrix formation (AMF) method (Aboanber and Nahla, 2007) is characterized by the expansion of the coefficients matrix for all mesh points of the reactor into two different

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matrices, specified for each mesh point. The generalization of the analytical exponential model GAEM (Nahla and Al-Ghamdi, 2012) is based on the separation of variables, the eigenvalues and the corresponding eigenvectors of the coefficient matrix. Fundamental matrix method (FMM) (Aboanber et al., 2014) is based on the separation of variables and the inverse of fundamental matrix.

Padé11 approximation and adaptive matrix formation (AMF) method are related to the proposed method, efficient computational system (ECS), in this work. The difference between Padé11 approximation, AMF and ECS methods is explained as follows: let us assume that n is the number of spatial mesh point, and m is the number of neutron energy groups and the number of delayed neutron precursors. Padé11 approximation is based on the inverse of a matrix $\mathbf{X}_{N\times N}$, where $N = n \times m$. The AMF method is based on the inverse of n times of a matrix $\mathbf{X}_{m\times m}$. But the proposed method ECS does not based on the inverse of any matrix. So, the proposed method ECS will be faster than AMF method which is faster than Padé approximation.

In this work, the space–time neutron diffusion equations with two energy groups and one group of delayed neutrons are introduced as partial differential equations in Section 2. These equations are reduced to ordinary differential equations using finite difference method. These ordinary differential equations are rewritten in a matrix form in Section 3. Theta method is presented to solve the matrix form of the ordinary differential equations in Section 4. The efficient computational system using the eigenvalues and corresponding eigenvectors of the coefficient matrix is presented in Section 5. These eigenvalues and corresponding eigenvectors are calculated analytically in Section 6. The efficient computational system is applied to solve two energy groups transient neutron diffusion equations with one group of delayed neutrons in multi-dimensional homogenous and heterogenous nuclear reactors in Section 7. The results of the proposed method are discussed and compared with the results of traditional methods. The general conclusions and future work are presented in Section 8.

2. Transient neutron diffusion model

The concern here is focused on the most common approximation of the time-dependent neutron diffusion equations with two energy groups and one group of delayed neutrons. The derivation of the neutron diffusion equations from the continuous energy transport equations is described in detail in a number of references (Duderstadt and Hamilton, 1976; Stacey, 2007; Baudron et al., 2014). The space–time two energy groups reactor kinetics equations can be written as

$$\frac{1}{v_1}\frac{\partial}{\partial t}\Phi_1(x,y,z,t) = \nabla\cdot D_1(x,y,z)\nabla\Phi_1(x,y,z,t) - (\Sigma_{a_1}(x,y,z) + \Sigma_{s_{12}}(x,y,z))\Phi_1(x,y,z,t) + (1-\beta)\nu_1\Sigma_{f_1}(x,y,z) \times \Phi_1(x,y,z,t) + (1-\beta)\nu_2\Sigma_{f_2}(x,y,z)\Phi_2(x,y,z,t) + \lambda C(x,y,z,t) \quad (1)$$

$$\frac{1}{v_2}\frac{\partial}{\partial t}\Phi_2(x,y,z,t) = \nabla\cdot D_2(x,y,z)\nabla\Phi_2(x,y,z,t) - \Sigma_{a_2}(x,y,z)\Phi_2(x,y,z,t) + \Sigma_{s_{12}}(x,y,z)\Phi_1(x,y,z,t) \quad (2)$$

Average one group of delayed precursor concentration satisfies the equation

$$\frac{\partial}{\partial t}C(x,y,z,t) = \beta\left[\nu_1\Sigma_{f_1}(x,y,z)\Phi_1(x,y,z,t) + \nu_2\Sigma_{f_2}(x,y,z)\Phi_2(x,y,z,t)\right] - \lambda C(x,y,z,t), \quad (3)$$

where, $\Phi_1(x,y,z,t)$ and $\Phi_2(x,y,z,t)$ are the fast and thermal neutron flux $(\mathrm{cm}^{-2}\mathrm{s}^{-1})$, $C(x,y,z,t)$ is the precursor concentration of delayed neutrons (cm^{-3}), ν_1 and ν_2 are the fast and thermal velocity of the neutron $(\mathrm{cm\,s}^{-1})$, $D_1(x,y,z)$ and $D_2(x,y,z)$ are the fast and thermal diffusion coefficient (cm), $\Sigma_{a_1}(x,y,z)$ and $\Sigma_{a_2}(x,y,z)$ are the fast and thermal absorption cross-section (cm^{-1}), $\Sigma_{f_1}(x,y,z)$ and $\Sigma_{f_2}(x,y,z)$ are the fast and thermal fission cross-section (cm^{-1}), $\Sigma_{s_{12}}(x,y,z)$ is the scattering cross-section (cm^{-1}) from fast to thermal, ν_1 and ν_2 are the fast and thermal neutrons emitted per fission, λ is the decay constant (s^{-1}), and β is the fraction of delayed neutrons.

Eqs. (1)–(3) are completed by the appropriate initial and boundary conditions. Regarding the boundary conditions, these equations are solved subject to the boundary conditions at the inner and outer surfaces of the reactor. The boundary condition can be written in general as

$$p_g D_g(x,y,z)\nabla\Phi_g(x,y,z,t).\mathbf{n} + q_g\Phi_g(x,y,z,t) = 0, \quad g = 1,2 \quad \forall(x,y,z)\in R. \quad (4)$$

where $\mathbf{n}$ is an outward unit normal to surface R of reactor. If q_g is equal to zero on part of the boundary, the normal neutron current flux $\mathbf{J}_g(x,y,z,t)$ is zero on that part. If p_g is equal to zero on part of the boundary, the corresponding neutron flux $\Phi_g(x,y,z,t)$ is equal to zero on that part. Finally, the general case p_g and $q_g \neq 0$ corresponds to an extrapolation condition.

At $t = 0$, the initial conditions satisfy

$$\Phi_g(x,y,z,0) = \Phi_g(x,y,z), \quad g = 1,2 \quad \text{and} \quad C(x,y,z,0) = \frac{\beta}{\lambda}\left[\frac{\nu_1\Sigma_{f_1}(x,y,z)}{k_{eff}}\Phi_1(x,y,z) + \frac{\nu_2\Sigma_{f_2}(x,y,z)}{k_{eff}}\Phi_2(x,y,z)\right] \quad (5)$$

The initial fast and thermal neutron flux usually correspond to a critical state obtained from Eqs. (1) and (2), with $\frac{\partial\Phi_g(x,y,z,t)}{\partial t} = 0$. The resulting equations, in which the spatial variables have been suppressed, are

$$\nabla\cdot D_1(x,y,z)\nabla\Phi_1(x,y,z) - \left(\Sigma_{a_1}(x,y,z) + \Sigma_{s_{12}}(x,y,z)\right)\Phi_1(x,y,z) + \frac{\nu_1\Sigma_{f_1}(x,y,z)}{k_{eff}}\Phi_1(x,y,z) + \frac{\nu_2\Sigma_{f_2}(x,y,z)}{k_{eff}}\Phi_2(x,y,z) = 0 \quad (6)$$

$$\nabla\cdot D_2(x,y,z)\nabla\Phi_2(x,y,z) - \Sigma_{a_2}(x,y,z)\Phi_2(x,y,z) + \Sigma_{s_{12}}(x,y,z)\Phi_1(x,y,z) = 0 \quad (7)$$

where k_{eff} is the effective fission multiplication factor.

This work presents efficient and accurate technique to solve these stiff partial differential equations. This technique is based on the finite difference method to reduce these partial differential equations to ordinary differential equations. Also, it depends on the theta method to solve the matrix form of the ordinary differential equations.

3. Finite difference method

The neutron leakage terms $\nabla\cdot D_g(i,j,k)\nabla\Phi_g(i,j,k,t), g = 1,2$ for three-dimensional Cartesian geometry are approximated using the seven-point central finite difference scheme (Aboanber and Nahla, 2007; Nahla et al., 2012)