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Methodology for modal analysis at pulsed neutron source experiments in accelerator-driven systems

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ABSTRACT

The evaluation of experimental techniques for the determination of the subcriticality level of an ADS core relies on reactor point kinetics. The measured parameters depend on the detector position, and the reactivity values are subject to spatial correction factors (SCFs). In this paper the contribution of different eigenmodes to the SCF for the pulsed neutron source (PNS) technique is assessed. Moreover by pulse simulations, precise values of the correction factor are obtained.

As case study, the VENUS-F SC1 subcritical core is investigated, with a pulsed neutron source in the center of the core, generated by the GENEPI-3C deuteron accelerator. Much more than 100 modes need to be taken into account to precisely obtain the spatial correction factor for the area method evaluation of this core. Especially the modes with a maximum in the center of the core contribute significantly to the SCF. No spatial correction needs to be applied close to the zeros of the first mode with a maximum in the center of the core (different from the fundamental one). In the reflector zone, the absolute reactivity level is overestimated.

1. Introduction

In an Accelerator-Driven System (ADS) (Nifenecker et al., 2003), accelerated particles create external source neutrons that drive a subcritical reactor core. Thanks to the subcriticality of the core, fuels with a small delayed neutron fraction can be operated with a large margin to prompt supercriticality. On the long term, the transmutation of long-lived minor actinides can be envisaged in ADS.

In order to guarantee a sufficient margin to criticality and a precise follow-up of the burn-up of the reactor fuel, the monitoring of subcriticality during all phases of operation of an ADS is indispensable. The MUSE project (Soule et al., 2004) comprised different experimental evaluations of subcriticality measurements, such as the pulsed neutron source (PNS) technique and the source jerk technique. All considered techniques are based on point kinetics reactor theory, however the presence of a local neutron source does not justify the use of point kinetics without correction factors. Therefore several types of so-called spatial correction factors (SCFs) exist, depending on the experimental techniques.

Modal analysis (Duderstadt and Hamilton, 1976) of SCFs helps in the understanding of the deviation from the point kinetics approximation of a subcritical experiment evaluation. Both dynamic α-modes (Lathouwers, 2003; Lathouwers, 2006; Kópházi and Lathouwers, 2012) and static λ-modes (Warsa et al., 2004) can be used to investigate the pulsed neutron source experiment. In this paper the so-called λ-mode analysis is applied for the PNS experiment evaluation, as this approach is less sensitive to kinetic parameters (Soule et al., 2004; Mellier, 2005). In the past, the λ-mode approach has been used to set up detector positioning for PNS experiments in an approximate way (Vandeplas, 1973).

For a clear understanding of the SCF, multi-group modal analysis is performed, and the contributions of different modes to the SCF are calculated. This unique approach allows the identification of important modes and positions where no spatial correction needs to be applied. Secondly, a precise calculation of the spatial correction factor is performed in an efficient way for the complete reactor core by simulation of the PNS experiment in the DALTON diffusion code (Boer et al., 2008, 2010). Contrary to probabilistic codes, the pulse-train build-up before the evaluation of the PNS

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experiment in equilibrium conditions does not need to be simulated thanks to the choice of initial conditions. This approach saves computational time.

In this paper first the area method (also called Sjöstrand method (Sjöstrand, 1956)) is presented to evaluate the subcriticality level of an ADS via the PNS experimental technique. Then the theory of static modal analysis is discussed and applied to reconstruct a fixed source problem solution by mode amplification. The multi-energy group modal SCF for the area method is determined to understand the spatial dependence. By using the DALTON diffusion code, eigenfunctions and their amplification coefficients are calculated for the VENUS-F subcritical reactor core SC1, modified towards an ADS within the framework of the GUINEVERE (Baeten et al., 2008) and FREYA project (Kochetkov, 2010). Finally, pulse simulations are made by DALTON to determine precisely the SCF for the PNS experiment in the VENUS-F SC1 core.

2. The area (Sjöstrand) method for the evaluation of the pulsed neutron source experiment

2.1. Description of the area method

The area method (also called Sjöstrand method) (Sjöstrand, 1956) is a static evaluation technique for subcriticality measurements by means of a pulsed neutron source. It states that the reactivity of a subcritical system driven by a pulse train of external source neutrons, can be determined by the ratio of two areas in the decay of the neutron density after a pulse, as shown in Fig. 1.

$$\frac{-\rho}{\beta} = \frac{A_p}{A_d} \tag{1}$$

with ρ the reactivity (in pcm), β the delayed neutron fraction (in pcm), A_p the area related to the prompt neutrons and A_d the area related to the delayed neutrons.

The area method makes use of point kinetics theory to obtain the expression (1) for the reactivity. Therefore, one assumes that the flux can be represented by a single, energy independent spatial mode, called fundamental mode (Duderstadt and Hamilton, 1976). In ADS however, a local external neutron source is introduced in a subcritical core and many modes are amplified to contribute to the total flux. Therefore spatial correction factors need to be applied on the area method reactivity level to correct for the point kinetic approach.

Using point kinetics the neutron density $n(t)$ after one Dirac pulse $S\delta(t-0)$ of an external neutron source with strength S (n/s) in a subcritical medium is given by (Baeten et al., 2006)

$$n(t) = S\left(exp\left(\frac{\rho-\beta}{\Lambda}t\right) + \frac{\lambda\beta\Lambda}{(\rho-\beta)^2}exp\left(\frac{\rho\lambda}{\rho-\beta}t\right)\right) \tag{2}$$

with Λ the neutron generation time (in s) and λ the average decay constant of the precursors ($\frac{1}{s}$). The theory can be easily expanded towards more families of delayed neutrons. For clarification only one family of delayed neutrons is shown in Eq. (2). One can distinguish the prompt and delayed contribution to the neutron density via the different exponential constants.

The area method is only valid when the asymptotic precursor concentration is reached. In (Baeten et al., 2006) the derivation of the neutron density is performed for an infinite pulse train $S = \sum_{n=0}^{\infty}\delta(t-nT)$ with pulse period T:

$$n(t) = S\left(exp\left(\frac{\rho-\beta}{\Lambda}t\right) + \frac{\beta\Lambda}{(\rho-\beta)\rho T}exp\left(\frac{\rho\lambda}{\rho-\beta}t\right)\right) \tag{3}$$

When integrating (3) over the period T and separating the prompt from the delayed contribution, (1) is obtained.

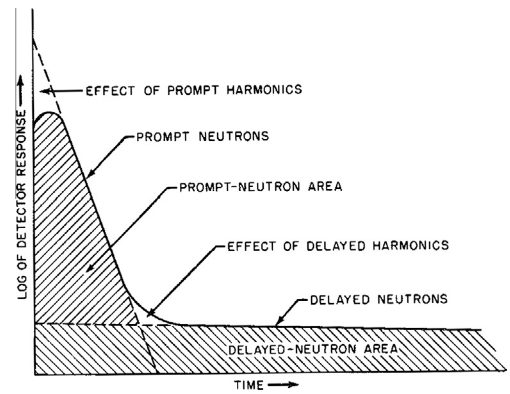

Fig. 1. Graphical representation of the PNS Area Evaluation Method (from (Bell and Glasstone, 1970)).

2.2. Spatial correction factors for the area method evaluation of the PNS experiment

The SCF for the area evaluation of the PNS experiment is defined as

$$f_{area} = \frac{\rho_c}{\frac{A_p^c}{A_d^c}} \tag{4}$$

with ρ_c the calculated reactivity value (in), A_p^c and A_d^c the simulated detector counts during a pulse at a detector location, associated to prompt and delayed neutrons, as in Eq. (1).

The most common way to calculate SCFs for the area method is to simulate the PNS experiment in a neutronic code. For probabilistic codes such as MCNP (X) (Pelowitz, 2011), this implies a complex procedure. As the asymptotic precursor concentration needs to be reached, many (in the order of 10^5–10^6, depending on the parameters of the PNS experiment) pulses need to be simulated before the area evaluation can be performed, which requires significant calculation time. For deterministic codes, this issue is tackled by the quasi-static initial conditions (see Section 4.3).

In the following paragraph the equation for the SCF will be derived by means of modal analysis. In this way one can explain the behaviour of the SCF throughout the volume of the reactor core. Moreover locations are identified where minimum spatial correction needs to be applied on the PNS area evaluation results.

3. Determination of optimum detector positioning via modal analysis of the spatial correction factor

3.1. λ-Eigenmodes

The behaviour of a nuclear reactor is determined by the distribution of the neutrons in the system in function of time, space, energy. The prediction of its behaviour is obtained by solving the transport equation or Boltzmann equation (Bell and Glasstone, 1985). In steady state and presented in operator form, this equation becomes

$$(F-L)\phi + S = 0 \tag{5}$$

with S the source strength, F the fission operator and L the transport operator comprising neutron leakage, neutron collisions, and neutron scattering:

$$F = \frac{\chi(E)}{4\pi}\int dE' \int d\widehat{\Omega}' \nu(E')\Sigma_f(\vec{r},E)\phi(\vec{r},\Omega',E') \tag{6}$$

$$\begin{aligned} L = {} & \widehat{\Omega}.\nabla\phi(\vec{r},\Omega,E) + \Sigma_t(\vec{r},\Omega,E) \\ & - \int dE' \int d\widehat{\Omega}'\Sigma_s(\vec{r},\Omega' \to \Omega, E' \to E)\phi(\vec{r},\Omega,E) \end{aligned} \tag{7}$$

$$S = S(\vec{r},\Omega,E) \tag{8}$$

using the typical notation as in (Bell and Glasstone, 1985).