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The Source Equivalence Acceleration Method

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ABSTRACT

Fine-group whole-core reactor analysis remains one of the long sought goals of the reactor physics community. Such a detailed analysis is typically too computationally expensive to be realized on anything except the largest of supercomputers. Recondensation using the Discrete Generalized Multigroup (DGM) method, though, offers a relatively cheap alternative to solving the fine group transport problem. DGM, however, suffered from inconsistencies when applied to high-order spatial methods. While an exact spatial recondensation method was developed and provided full spatial consistency with the fine group problem, this approach substantially increased memory requirements for realistic problems. The method described in this paper, called the Source Equivalence Acceleration Method (SEAM), forms a coarse-group problem which preserves the fine-group problem even when using higher order spatial methods. SEAM allows recondensation to converge to the fine-group solution with minimal memory requirements and little additional overhead. This method also provides for consistency when using different spatial methods and angular quadratures between the coarse group and fine group problems. SEAM was implemented in OpenMOC, a 2D MOC code developed at MIT, and its performance tested against Coarse Mesh Finite Difference (CMFD) acceleration on the C5G7 benchmark problem and on a 361 group version of the problem. For extremely expensive transport calculations, SEAM was able to outperform CMFD, resulting in speed-ups of 20–45 relative to the normal power iteration calculation.

1. Introduction

Achieving high fidelity neutronics analysis is of utmost importance in analyzing the performance and safety of reactor designs. Since quantities such as power density are dependent on the neutron distribution, accurate predictions of the neutron flux in a reactor are key to performing steady-state and transient safety analyses. These analyses are conducted by solving the neutron transport equation.

Ideally, this equation would be solved exactly for a given 3D reactor model including a detailed representation of all reactor internals and their corresponding cross section data. Such solutions are very expensive computationally especially when one tries to reduce the approximations introduced in the energy condensation process which can require up to 1000's of energy groups. It is thus imperative to make proper use of low-order acceleration models to reduce the computational cost and provide a platform for fluids and fuel performance coupling. The current state-of-the-art in acceleration usually involves some form of non-linear acceleration (Kim and DeHart, 2011; Park and Cho, 2004; Smith, 1983; Yamamoto, 2005; Zhong and et al., 2008) which focuses on spatial acceleration for high dominance ratio cores or the generalized multigroup method with recondensation, a non-linear approach that aims at high dimension energy problems (Douglass and Rahnema, 2012; Rahnema et al., 2008; Zhu and Forget, 2010, 2011). One of the main problems with generalized multigroup and recondensation is that the nonlinear iteration is not consistent with the fine group problem when using high order spatial methods, meaning the nonlinear iteration does not converge to the fine group solution (Everson and Forget, 2013; Everson, 2014). Previous work modifying the generalized multigroup equations produced full consistency with the fine group problem but came with a substantial increase in memory requirements. To overcome this limitation, this paper proposes a new nonlinear acceleration method as the primary means of producing high fidelity solutions in an efficient manner for high energy and angular dimensionality problems.

2. Review of nonlinear acceleration

Nonlinear acceleration methods rely on an iterative approach in which full transport sweeps are used to create an equivalent yet

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cheaper version of the problem. These cheaper versions can be formed using spatial homogenization, energy condensation and/or a low order angular approximation. The solutions from these equivalent problems are then used to reduce the total number of full transport sweeps required to converge the fission source. This provides a relatively cheap process by which the true solution of the full problem can be obtained.

2.1. Coarse Mesh Finite Difference diffusion and Coarse Mesh Rebalance

Coarse Mesh Finite Difference (CMFD) diffusion is a widely used nonlinear acceleration scheme in reactor physics (Kim and DeHart, 2011; Smith, 1983; Zhong and et al., 2008). In this approach, the high order angular problem is represented by a low-order diffusion problem. Equivalence is enforced through calculation of nonlinear diffusion coefficient correction factors (diffusion correction factors for short). Equivalence can be maintained across spatial homogenization and energy condensation. The diffusion correction factors are calculated according to Eq. (2.1) when incorporating energy condensation simultaneously with spatial homogenization.

$$\widetilde{D}_{c+1/2,g} = -\frac{J_{c+1/2,g} + \widehat{D}_{c+1/2,g}\left(\phi_{c+1,g} - \phi_{c,g}\right)}{\phi_{c+1,g} + \phi_{c,g}} \tag{2.1}$$

Definitions for each quantity are provided below:

- $\phi_{c,g}$ is the scalar flux for coarse mesh cell c and coarse group g.
- $\widehat{D}_{c+1/2,g}$ is the effective diffusion coefficient between coarse mesh cells c and $c+1$ for coarse group g.
- $J_{c+1/2,g}$ is the net surface current across the surface spanning coarse mesh cells c and $c+1$ for coarse group g.

The procedure highlighted in Eq. (2.1) takes into account energy condensation to create a cheaper version of a fine group transport calculation through the simultaneous spatial homogenization and multigroup collapse of scattering, fission, total cross sections, diffusion coefficients and scalar fluxes (Lee, 2012). An example of this procedure is provided for the fission cross section in Eq. (2.2).

$$\nu\Sigma_{F,c,g} = \sum_{K\in g}\sum_{f\in c}\nu\Sigma_{F,f,K}\phi_{f,K}d_f \Bigg/ \sum_{K\in g}\sum_{f\in c}\phi_{f,K}d_f \tag{2.2}$$

The summations take place across the individual fine groups within a coarse group g and are indexed using K. f is used to denote each of the fine mesh cells homogenized into coarse mesh cell c. d_f is the length of fine mesh cell f in a 1D problem.

Once the diffusion correction factors are calculated for each surface, a set of finite-difference diffusion equations are solved to provide the new fluxes for each of the coarse meshes. The converged coarse mesh fluxes are then used to update the fine mesh fine group fluxes, thereby accelerating the transport problem.

An alternative to CMFD is the Coarse Mesh Rebalance (CMR) method (Park and Cho, 2004; Yamamoto, 2005). In this case, neutron conservation is maintained directly through the neutron balance equation by multiplying the surface currents at coarse mesh cell boundaries by a set of rebalance factors (Hong et al., 2010).

Application to a 1D problem is shown in Eq. (2.3) with energy condensation included.

$$\left(J^{+}_{c,g} + J^{-}_{c,g} + \sum_{f\in c} d_f\Sigma_{T,f,g}\phi_{f,g}\right)R_{c,g} - J^{+}_{c-1,g}R_{c-1,g} - J^{-}_{c+1,g}R_{c+1,g} = \sum_{f\in c} d_f Q_{c,g} \tag{2.3}$$

Definitions for each quantity are provided below:

- $J^{+/-}_{c,g}$ is partial surface current for coarse group g exiting coarse mesh cell c from the right and left surfaces, respectively.
- $R_{c,g}$ is the rebalance factor for coarse mesh cell c and coarse group g.
- $Q_{c,g}$ is the source for coarse mesh cell c and coarse group g.

Eq. (2.3) forms a series of equations from which the rebalance factors can be solved. The rebalance factors are then used to update the fine mesh fine group cell-averaged scalar fluxes and accelerate the transport problem.

However, it is important to note that both CMFD and CMR suffer from conditional stability issues. Both methods suffer from instabilities for optically thick meshes, but only CMFD appears to be stable for optically thin coarse meshes (Kim and DeHart, 2011). If CMFD is able to converge for an optically thick problem, little to no acceleration is observed (Lee, 2012). In many applications of CMFD, a dampening factor is applied to prevent instabilities from growing and causing divergence. The dampening factor prevents overshoot of the accelerated scalar fluxes by limiting how quickly the nonlinear diffusion coefficient correction factor from Eq. (2.1) is updated. This damping factor is largely problem dependent but a reasonable value can be applied across a number of different problems (Boyd et al., 2014). Partial current Coarse Mesh Finite Difference (pCMFD) is a more recent development which has also been shown to improve stability. This approach adds a second degree of freedom to CMFD by calculating two partial current corrective factors at the boundaries of a coarse mesh instead of forming a single net current corrective factor (Kim and DeHart, 2011; Hong et al., 2010).

While multigroup collapse can be and has been incorporated into CMFD and CMR, the primary focus of acceleration for both methods is typically placed on the space-angle problem.

2.2. Exact recondensation

DGM differs from these methods in that its primary focus is on accelerating the spectral problem. This approach is essentially a discrete form of the Generalized Energy Condensation (GEC) method, which was the first method to apply a transformation to the original fine group equations using a series of continuous polynomial functions in energy (Rahnema et al., 2008). On the other hand, DGM uses discrete basis functions to represent the fine group angular flux as angular flux moments according to Eq. (2.4) (Zhu and Forget, 2010). In this equation, $\psi_K\left(\vec{r},\vec{\Omega}\right)$ is the angular flux for fine group K in coarse group g, $P_{i,K}$ is the i^{th} discrete basis function evaluated at fine group K and $\psi_{i,g}\left(\vec{r},\vec{\Omega}\right)$ is the i^{th} angular flux moment in coarse group g.

$$\psi_{i,g}\left(\vec{r},\vec{\Omega}\right) = \sum_{K\in g} P_{i,K}\psi_K\left(\vec{r},\vec{\Omega}\right) \tag{2.4}$$

The use of discrete basis functions to represent the fine group fluxes better matches the discrete nature of the cross section data. The fine group angular flux can be expanded in terms of these angular flux moments within the multigroup transport equation using Eq. (2.5). a_i is the normalization constant for the i^{th} discrete basis function and N_g is the total number of fine groups in coarse group g, which is equivalent to the total number of moments in coarse group g.

$$\psi_K\left(\vec{r},\vec{\Omega}\right) = \sum_{i=0}^{N_g-1} a_i P_{i,K}\psi_{i,g}\left(\vec{r},\vec{\Omega}\right) \tag{2.5}$$