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The modified exponential time differencing (ETD) method for solving the reactor point kinetics equations

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ABSTRACT

The exponential time differencing (ETD) method with Taylor's series approximation is developed to solve the reactor point kinetics equations using large time-step. It is a semi-analytical and self-starting method in which the point kinetics equations are integrated using an integrating factor. The power and precursor concentrations occurring in the equations are expanded as a polynomial in derivatives. The coefficients of the polynomial are obtained from the recurrence relation. This method is applied to estimate the power transients of both thermal and fast reactors with multi group of delayed neutrons and the results are compared with other standard methods. From the results it is found that the modified exponential time differencing method estimates the power transients accurately for thermal and fast reactors and the numerical solution is found to converge fast. The error analysis of this method is discussed and a criterion for choosing the time-step for step and slowly varying reactivity insertions is also presented.

1. Introduction

The power transients taking place in nuclear reactors depend on the kind of reactivity perturbation acting on the reactor and also the power level in which the reactor is operating. It is important to predict the power transients, caused by reactivity perturbations, during normal operation and accidental conditions. Though the space time kinetics represents the time-dependent behaviour of nuclear reactor in a more realistic way, solving the space time kinetics equations consumes more time. Determination of spatial variation of flux in a reactor is more difficult than the determination of power. So, generally in medium sized thermal reactors and tightly coupled fast reactors, the point kinetics is found to be satisfactory in predicting the power transients. The point kinetics equations with multi group of delayed neutrons are stiff non-linear differential equations. A major difficulty in numerically solving the point kinetics equations arises from the stiffness term, which necessitates the requirement of choosing very small time-step. There are different kinds of methodologies available in the literature to study the reactor transients. Hennart and Barrios (1976) applied the Pade and Chebyshev type approximation to solve the point kinetics equations. One can use the third order Hermite Polynomial Method (HPM) (Yeh, 1978) to solve the point kinetics equations. Aboanber and Nahla (2002a,b) presented the analytical inversion method for solving the point reactor kinetics equations with multi group of delayed neutrons. Li et al. (2009) introduced a numerical integral method using the better basis function (BBF) and investigated the power transients caused by various types of reactivity insertion for thermal reactors with multi group of delayed neutrons. Recently Nahla (2011) applied the Taylor's series method (TSM) to solve the point kinetics equations.

In this work, the exponential time differencing (ETD) method with Taylor's series expansion is developed and presented for solving the point kinetics equations with multi group of delayed neutrons. The exponential time differencing (ETD) method was originally proposed in the field of computational electrodynamics (Taflove, 1995) to solve stiff differential equations.

This method is developed by integrating the point kinetics equations over a single time-step using an integrating factor. The power and precursor concentrations in the equations are expanded as a polynomial in derivatives and then integrated. The coefficients of the polynomial (ETD coefficients) are obtained from integration of simple function and a recurrence relation between them is also obtained. The recurrence relation is used to determine higher order ETD coefficients. This method is applied to estimate the power transients of thermal reactors and the results are compared with other standard methods referred by Nahla (2011) and Li et al. (2009). This method is also applied to estimate the power transients of the Indian Prototype Fast

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Breeder Reactor (PFBR) at IGCAR Kalpakkam and the results are compared with the Cohen's method (1958). From the comparison of results, it is learnt that the modified ETD method estimates the power transients accurately using large time-step for thermal and fast reactors with multi group of delayed neutrons. The results also show that the power transients may be estimated for longer duration of time. The error analysis of this method is discussed and the numerical solution is found to converge fast as the order of the ETD coefficients is increased. From the error analysis, a novel way of choosing the time-step, 'h' and its upper bound for convergence of the numerical solution are also derived and presented.

2. The modified exponential time differencing (ETD) method and the point kinetics equations

The exponential time differencing (ETD) methods are time integration techniques that provide accurate smooth solutions for stiff differential equations.

Consider the following stiff differential equation

$$\frac{du}{dt} = cu + F(u,t) \tag{1}$$

In the above equation 'c' is the stiffness constant which may be large and $F(u,t)$ may be a nonlinear term. To numerically solve this kind of stiff differential equation, one should be able to handle the stiffness constant 'c', by properly choosing the time-step 'h'. To solve Eq. (1) using the exponential time differencing method, it is multiplied by an integrating factor e^{-ct} and integrated over a single time-step, from $t = t_n$ to $t = t_{n+1} = t_n + h$, to get

$$\int_{t_n}^{t_{n+1}} \frac{d}{dt}(ue^{-ct})dt = \int_{t_n}^{t_{n+1}} F(u,t)e^{-ct}dt \tag{2}$$

After integrating, we get

$$u(t_{n+1}) = u(t_n)e^{ch} + e^{ch}\int_0^h e^{-c\tau}F(u(t_n+\tau), t_n+\tau)d\tau \tag{3}$$

where $\tau = t - t_n$. The purpose of transforming the differential Eq. (1) into Eq. (2) is to remove the explicit dependence of the stiffness constant 'c' in Eq. (1). In the numerical solution, i.e., Eq. (3), the stiffness constant appears only in the exponential term. By removing the explicit dependence of the stiffness constant, the solution, Eq. (3), may be obtained using large time-step 'h'. Another major critical step in Eq. (3) lies in choosing a proper approximation for the integrand $F(u,t)$ in the interval $t_n < t < t_n + h$. If $F(u,t)$ is known a priori for $t < t_n$, then $F(u,t)$ in the interval $t_n < t < t_n + h$ may be obtained using interpolation with its previous values and the integral in Eq. (3) can be evaluated (Beylkin et al., 1998;Nie et al., 2006). In case if $F(u,t)$ is not known a priori, then $F(u,t)$ in the time interval $t_n < t < t_n + h$ can be obtained from the Taylor's series expansion of $F(u,t)$.

Consider the point kinetics equations given by

$$\frac{dp(t)}{dt} = \left(\frac{\rho(t)-\beta}{\wedge}\right)p(t) + \sum_{i=1}^{6}\lambda_i C_i(t) \tag{4}$$

$$\frac{dC_i}{dt} = \left(\frac{\beta_i}{\wedge}\right)p(t) - \lambda_i C_i \quad (i = 1,2\ldots 6) \tag{5}$$

In the above Eqs. (4) and (5), p is the power, $\wedge$ is the prompt neutron generation time, β_i is the effective fraction of the i^{th} group of delayed neutrons, β is the total effective fraction of delayed neutrons $\left(\beta = \sum_{i=1}^{6}\beta_i\right)$, λ_i and C_i are the decay constant and precursor concentration of the i^{th} group of delayed neutron. The initial conditions of the point kinetics equations are chosen as $p(t=0) = p_0$, $c_i(t=0) = \frac{\beta_i}{\wedge\lambda_i}p_0$, p_0 is the steady state power before the introduction of any external reactivity. The neutron generation time is of the order of $\sim 10^{-5}$ s for thermal reactors and it is of the order of $\sim 10^{-7}$ s for fast reactors. Here the stiffness constant appears as $\frac{1}{\wedge}$. To solve the point kinetics equations by the modified ETD method, Eqs. (4) and (5) are multiplied by the integrating factor $e^{\frac{t}{\wedge}}$ and integrated over a single time-step from $t = t_n$ to $t = t_{n+1} = t_n + h$, to get

$$\int_{t_n}^{t_{n+1}} \frac{d}{dt}\left(pe^{\frac{t}{\wedge}}\right)dt = \int_{t_n}^{t_{n+1}} \left(\frac{\rho(t)-\beta}{\wedge}\right)pe^{\frac{t}{\wedge}}dt + \int_{t_n}^{t_{n+1}}\sum_{i=1}^{6}\lambda_i C_i(t)e^{\frac{t}{\wedge}}dt + \frac{1}{\wedge}\int_{t_n}^{t_{n+1}} pe^{\frac{t}{\wedge}}dt \tag{6}$$

$$\int_{t_n}^{t_{n+1}} \frac{d}{dt}\left(C_i e^{\frac{t}{\wedge}}\right)dt = \int_{t_n}^{t_{n+1}} \frac{\beta_i}{\Lambda}pe^{\frac{t}{\wedge}}dt + \left(\frac{1}{\wedge} - \lambda_i\right)\int_{t_n}^{t_{n+1}} C_i(t)e^{\frac{t}{\wedge}}dt \tag{7}$$

After integrating and rearranging Eqs. (6) and (7), we get

$$p(t_{n+1}) = p(t_n)e^{\frac{-h}{\wedge}} + \int_0^h e^{\frac{\tau-h}{\wedge}}\frac{\rho(t_n+\tau)}{\wedge}p(t_n+\tau)d\tau + \left(\frac{1-\beta}{\wedge}\right)\int_0^h e^{\frac{\tau-h}{\wedge}}p(t_n+\tau)d\tau + \int_0^h e^{\frac{\tau-h}{\wedge}}\sum_{i=1}^{6}\lambda_i C_i(t_n+\tau)d\tau \tag{8}$$

$$C_i(t_{n+1}) = C_i(t_n)e^{\frac{-h}{\wedge}} + \frac{\beta_i}{\Lambda}\int_0^h e^{\frac{\tau-h}{\wedge}}p(t_n+\tau)d\tau + \left(\frac{1}{\wedge} - \lambda_i\right)\int_0^h e^{\frac{\tau-h}{\wedge}}C_i(t_n+\tau)d\tau \tag{9}$$

Here, in the above Eqs. (8) and (9), we expand $p(t_n+\tau)$ and $C_i(t_n+\tau)$ as a polynomial in derivatives using Taylor's series expansion of order '$S-1$'. They are expanded as

$$p(t_n+\tau) = \sum_{k=0}^{S-1}\nabla^k p(t_n)\frac{\tau^k}{k!} \tag{10}$$

$$C_i(t_n+\tau) = \sum_{k=0}^{S-1}\nabla^k C_i(t_n)\frac{\tau^k}{k!} \tag{11}$$

where $\nabla^k = \frac{d^k}{dt^k}$. The reason for expanding the power and precursor concentrations using Taylor's series is that they require only present values and previous power history is not required. The local truncation errors in the Taylor's series expansion of $p(t_n+\tau)$ and $C_i(t_n+\tau)$ are termed as $R_1(p)$ and $R_2(C_i)$ respectively and they are given as

$$R_1(p) = \sum_{k=S}^{\infty}\nabla^k p(t_n)\frac{\tau^k}{k!} \tag{12}$$

$$R_2(C_i) = \sum_{k=S}^{\infty}\nabla^k C_i(t_n)\frac{\tau^k}{k!} \tag{13}$$

Substituting Eqs. (10) and (11) into Eqs. (8) and (9), we get

$$p(t_{n+1}) = p(t_n)e^{\frac{-h}{\wedge}} + \frac{1}{\wedge}\int_0^h e^{\frac{\tau-h}{\wedge}}\rho(t_n+\tau)\left(\sum_{k=0}^{S-1}\nabla^k p(t_n)\frac{\tau^k}{k!}\right)d\tau + \left(\frac{1-\beta}{\wedge}\right)\int_0^h e^{\frac{\tau-h}{\wedge}}\left(\sum_{k=0}^{S-1}\nabla^k p(t_n)\frac{\tau^k}{k!}\right)d\tau + \sum_{i=1}^{6}\lambda_i\int_0^h e^{\frac{\tau-h}{\wedge}}\left(\sum_{k=0}^{S-1}\nabla^k C_i(t_n)\frac{\tau^k}{k!}\right)d\tau \tag{14}$$