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Perturbation based Monte Carlo criticality search in density, enrichment and concentration

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ABSTRACT

Criticality search is a very important aspect in reactor physics analysis. Due to the advantages of Monte Carlo method and the development of computer technologies, Monte Carlo criticality search is becoming more and more necessary and feasible. Existing Monte Carlo criticality search methods need large amount of individual criticality runs and may have unstable results because of the uncertainties of criticality results. In this paper, a new perturbation based Monte Carlo criticality search method is proposed and discussed. This method only needs one individual criticality calculation with perturbation tallies to estimate k_{eff} changing function using initial k_{eff} and differential coefficients results, and solves polynomial equations to get the criticality search results. The new perturbation based Monte Carlo criticality search method is implemented in the Monte Carlo code RMC, and criticality search problems in density, enrichment and concentration are taken out. Results show that this method is quite inspiring in accuracy and efficiency, and has advantages compared with other criticality search methods.

1. Introduction

Criticality control is very important in reactor design and operation, which is relied on criticality control ways, such as changing boron concentration, control rods position and so on. Therefore, criticality search, or eigenvalue search, is a very important aspect in reactor physics analysis. The Monte Carlo method can represent the neutron transport problem with minimal approximations in geometries and materials, and also has very good performances in parallel calculations. With the development of computer technologies, Monte Carlo method is used more and more widely in reactor analysis and design. According to the advantages of Monte Carlo method and the development of computer technologies, Monte Carlo criticality search is becoming more and more necessary and feasible.

Monte Carlo criticality search methods and capabilities have been researched and developed in some Monte Carlo codes, such as MC21 (Morrow et al., 2007). Existing Monte Carlo criticality search methods are based on running serial criticality calculations by adjusting the concerned parameter in the system to determine an acceptable k_{eff}. Obviously, this kind of methods needs several individual criticality runs in searching process, which will cost large amount of calculation time. In MC21, several techniques, such as Adaptive Batching Algorithm (ABA) (Morrow et al., 2007; Gill et al., 2013), have been implemented to improve the searching efficiency. Also, in searching processes, when the k_{eff} is close to target value, it may get unstable results because of the uncertainties and fluctuations of criticality results, which brings difficulties to determine convergence and final results. To overcome this problem, MC21 also defined the target k_{eff} area (Morrow et al., 2007; Gill et al., 2013), which has considered the effects of k_{eff} uncertainties to avoid unstable results.

In this paper, a new perturbation based Monte Carlo criticality search method is proposed and discussed. This method uses a quite different idea from other Monte Carlo criticality search methods, and only needs one individual criticality run with perturbation tallies and gives accurate criticality search results. In Section 2, the perturbation based Monte Carlo criticality search method is introduced, including the description of the perturbation calculation method and criticality search method, and the discussion with other criticality search methods. In Section 3, some numerical results are given to show the capability and feasibility of this method to solve criticality search problems with density, enrichment and concentration changes.

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2. Algorithms description

The eigenvalue k_{eff} can be considered as a function of concerned parameter, presented as:

$$k_{eff} = f(a)$$

in which, a is the concerned parameter which needs to be adjusted to make the system critical; $f(a)$ is the changing function of eigenvalue k_{eff}. Essentially, the criticality search is to solve the equation $f(a) = 1.0$, and get the critical value of parameter a.

Instead of taking serial criticality calculations in criticality search process like other criticality search methods, the perturbation based Monte Carlo criticality search method proposed in this paper uses perturbation tallies got from one criticality calculation to estimate the eigenvalue changing function $f(a)$ as $\tilde{f}(a)$, such as an approximated function or truncated Taylor function, and then solves the equation $\tilde{f}(a) = 1.0$ to get the critical value of parameter a, which is the result of criticality search calculation.

2.1. Perturbation calculation

There are several kinds of Monte Carlo perturbation methods, such as differential operator sampling method (Rief, 1994), correlated sampling method (Jerome and Ely M, 1969; Nakagawa and Asaoka, 1978; Kitada et al., 1996), adjoint weighted sampling method (Kiedrowski et al., 2011; Shim and Kim, 2011), and so on. Unfortunately, not all these methods can be used for criticality search calculation. The Monte Carlo perturbation calculation method which can be used for criticality search should meet the following requirements:

- The method could estimate the k_{eff} changing function $f(a)$ by one individual calculation with perturbation tallies;
- The method should get good perturbation results in a certain wide range of concerned parameter a.

According to these requirements, only the higher order differential operator sampling method with perturbed source effect can be used for the perturbation calculation in criticality search calculation.

In differential operator sampling method, the change of k_{eff} can be expressed with the Taylor series expansion as

$$\Delta k = \frac{\partial k}{\partial a}\Delta a + \frac{1}{2}\cdot\frac{\partial^2 k}{\partial a^2}(\Delta a)^2 + \ldots + \frac{1}{n!}\cdot\frac{\partial^n k}{\partial a^n}(\Delta a)^n + \ldots \quad (1)$$

in which, Δk is the change of k_{eff}, Δa is the change of concerned parameter a, such as the change of material density, nuclide concentration, geometry position, and so on. The differential operator sampling method estimates each of these differential coefficients in Eq. (1).

2.1.1. First order differential coefficient

In non-analog Monte Carlo simulation, the k_{eff} of i_{th} generation can be expressed as (Lux and Koblinger, 1991; Nagaya and Mori, 2005)

$$k_i = \frac{1}{\int dPS_{f,i}(P)}\sum_{m=1}^{\infty}\int dP\ldots\int dP_1\int d\vec{r}_0\times\left[\sum_{l=1}^{m}W_f(P_l,\ldots,P_1,\vec{r}_0)\right]$$
$$\times\,\tilde{\alpha}(P_m)\tilde{K}(P_m;P_{m-1})\ldots\tilde{K}(P_2;P_1)\tilde{T}(P_1;\vec{r}_0)\tilde{S}_{f,i}(\vec{r}_0,E_1,\vec{\Omega}_1) \quad (2)$$

in Eq. (2), P is the six-dimension state parameter of $(\vec{r},\vec{\Omega},E)$ to present the position, direction and energy of neutron; $\tilde{K}$ and $\tilde{S}_{f,i}$ are the biased transport kernel and fission source in non-analog Monte Carlo simulation, and W_f is defined as:

$$W_f(P_l,\ldots,P_1,\vec{r}_0) = \frac{\nu\Sigma_f(P_l)}{\Sigma_t(P_l)}W(P_l,\ldots,P_1,\vec{r}_0)$$

in which, ν is the average fission neutron number, Σ_t and Σ_f are the macro total cross-section and fission cross-section.

$W(P_l,\ldots,P_1,\vec{r}_0)$ is particle weight in the l_{th} collision, and it's the estimate for k_{eff}, which means k_i can be estimated by

$$Est[k_i] = \frac{1}{N}\sum_n w_{f,n}$$

in which, $w_{f,n}$ is the score for W_f. From Eq. (2), the first order differential coefficient can be calculated (Nagaya and Mori, 2005), it can be expressed as:

$$\frac{\partial k_i}{\partial a} = \frac{1}{\int dPS_{f,i}(P)}\times\sum_{m=1}^{\infty}\int dP_m\ldots\int dP_1$$
$$\times\int d\vec{r}_0\left[\sum_{l=1}^{m}W_f^{(1)}(P_l,\ldots,P_1,\vec{r}_0)W_f(P_l,\ldots,P_1,\vec{r}_0)\right]$$
$$\cdot\,\tilde{\alpha}(P_m)\tilde{K}(P_m;P_{m-1})\ldots\tilde{K}(P_2;P_1)\tilde{T}(P_1;\vec{r}_0)\tilde{S}_{f,i}(\vec{r}_0,E_1,\vec{\Omega}_1) \quad (3)$$

$$W_f^{(1)}(P_l,\ldots,P_1,\vec{r}_0) = \frac{\Sigma_t(P_l)}{\nu\Sigma_f(P_l)}\frac{\partial}{\partial a}\left(\frac{\nu\Sigma_f(P_l)}{\Sigma_t(P_l)}\right)$$
$$+\frac{1}{K(P_l;P_{l-1})}\frac{\partial}{\partial a}K(P_l;P_{l-1})+\ldots$$
$$+\frac{1}{K(P_2;P_1)}\frac{\partial}{\partial a}K(P_2;P_1)+\frac{1}{T(P_1;\vec{r}_0)}\frac{\partial}{\partial a}T(P_1;\vec{r}_0)$$
$$+\frac{1}{S_{f,i}(\vec{r}_0,E_1,\Omega_1)}\frac{\partial}{\partial a}S_{f,i}(\vec{r}_0,E_1,\Omega_1) \quad (4)$$

in which, W_f is the particle weight and $W_f^{(1)}$ is the additional weight for first order differential coefficient, K and S are the transport kernel and fission source. From Eq. (3), it shows that the first order differential coefficient of k_i can be estimated by scoring $W_f^{(1)}W_f$ at each collision site. Also, we can see from Eq. (4), the first order differential coefficient can be split into two parts, one is calculated without perturbed source effect and the other one is related with perturbed source effect, which can be expressed as Eq. (5):

$$\frac{\partial k_i}{\partial a} = \frac{\partial k_{NPS,i}}{\partial a}+\frac{\partial k_{PS,i}}{\partial a} = \frac{1}{\int dPS_{f,i}(P)}\sum_{m=1}^{\infty}\int dP_m\ldots\int dP_1$$
$$\int d\vec{r}_0\left[\sum_{l=1}^{m}(W_{f,NPS}^{(1)}+W_{f,PS}^{(1)})W_f(P_l,\ldots,P_1,\vec{r}_0)\right]$$
$$\cdot\,\tilde{\alpha}(P_m)\tilde{K}(P_m;P_{m-1})\ldots\tilde{K}(P_2;P_1)\tilde{T}(P_1;\vec{r}_0)\tilde{S}_{f,i}(\vec{r}_0,E_1,\vec{\Omega}_1) \quad (5)$$

In Eq. (5), $W_{f,NPS}^{(1)}$ and $W_{f,PS}^{(1)}$ are the additional weight for unperturbed source effect and perturbed source effect of the first order differential coefficient, which can be defined as Eqs. (6) and (7):

$$W_{f,NPS}^{(1)} = \frac{\Sigma_t(P_l)}{\nu\Sigma_f(P_l)}\frac{\partial}{\partial a}\left(\frac{\nu\Sigma_f(P_l)}{\Sigma_t(P_l)}\right)+\frac{1}{K(P_l;P_{l-1})}\frac{\partial}{\partial a}K(P_l;P_{l-1})+\ldots$$
$$+\frac{1}{K(P_2;P_1)}\frac{\partial}{\partial a}K(P_2;P_1)+\frac{1}{T(P_1;\vec{r}_0)}\frac{\partial}{\partial a}T(P_1;\vec{r}_0) \quad (6)$$

$$W_{f,PS}^{(1)} = \frac{1}{S_{f,i}(\vec{r}_0,E_1,\Omega_1)}\frac{\partial}{\partial a}S_{f,i}(\vec{r}_0,E_1,\Omega_1) \quad (7)$$

Therefore, from the above equations, we can get that the first order differential coefficient of k_i for unperturbed source system can be estimated by

$$Est\left[\frac{\partial k_{NPS,i}}{\partial a}\right] = \frac{1}{N}\sum_n w_{f,NPS,n}^{(1)}w_{f,n}$$

in which, $w_{f,NPS,n}^{(1)}$ is the score for $W_{f,NPS}^{(1)}$. Also, the first order differential coefficient for perturbed source effect can be estimated by

$$Est\left[\frac{\partial k_{PS,i}}{\partial a}\right] = \frac{1}{N}\sum_n w_{f,PS,n}^{(1)}w_{f,n}$$