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Point kinetics equations for subcritical systems based on the importance function associated to an external neutron source

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ABSTRACT

This paper aims to determine the parameters for a new set of equations of point kinetic subcritical systems, based on the concept of importance of Heuristic Generalized Perturbation Theory (HGPT). The importance function defined here is related to both the subcriticality and the external neutron source worth (which keeps the system at steady state). The kinetic parameters defined in this work are compared with the corresponding parameters when adopting the importance functions proposed by Gandini and Salvatores (2002), Dulla et al. (2006) and Nishihara et al. (2003). Furthermore, the point kinetics equations developed here are solved for two different transients, considering the parameters obtained with different importance functions. The results collected show that there is a similar behavior of the solution of the point kinetics equations, when used with the parameters obtained by the importance functions proposed by Gandini and Salvatores (2002) and Dulla et al. (2006), specially near the criticality. However, this is not verified as the system gets farther from criticality.

1. Introduction

Most of the commercial nuclear thermal reactors operate in an open fuel cycle, i.e., the fuel is not recycled. This generates an accumulation of spent nuclear fuel that, besides the valuable uranium and plutonium, contains also minor actinides (americium, curium and neptunium) and fission products of long half-life and extremely radiotoxic. This fuel needs to be stored safely for thousands of years and their management is considered to be one of the major challenges of nuclear energy.

In this context, the subcritical source-driven reactors appear as a plausible solution to this problem, since these systems have been recognized worldwide as a potential highly relevant tool in order to transmute very large amounts of radioactive wastes (Gandini and Salvatores, 2002).

These systems, commonly known by the acronym ADS (*Accelerator Driven Systems*), although initially dedicated exclusively for transmutation of these wastes in order to reduce their high activities and as a result, decrease the minimum requirements in geological repositories, can be used for power generation as they can extract an amount of energy equivalent to 30% of what is extracted from the initial burning in a PWR (Rubbia, 1996).

The ADS reactors consist of a subcritical system maintained at steady-state by an external neutron source. The presence of an external source of neutrons introduces new considerations with respect to the mathematical description of the physical process that occurs in the core of the nuclear reactor (Dulla et al., 2006) as in the case of point kinetics.

The set of point kinetics equations for critical systems is usually derived from the neutron transport theory, using the adjoint flux $\varphi_0^*(\vec{r}, E, \widehat{\Omega})$ as a weighting function, since the adjoint flux is the importance associated with the criticality.

However, subcritical systems such as the ADS cannot have its formulation based on the use of the adjoint flux as a weighting function, as it is not an adequate function for subcritical systems.

The main difficulty for the complete physical description of subcritical systems lies in the choice of the importance function associated to a given integral quantity (Dulla et al., 2006). In the literature, the works of the major highlights are those proposed by Gandini and Salvatores (2002), that takes into account the presence of external sources and fission and the concept of generalized reactivity, Dulla et al. (2006), which has the same structure as the classic point kinetic model, Nishihara et al. (2003), which does not account for the presence of fission source to calculate the importance function, and lastly, Silva et al. (2012), which is based on the choice of a hybrid importance function.

This paper presents a new formulation for the point kinetics of subcritical systems based on the choice of an importance function

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that is associated with both the subcriticality of the system as to the value of the external neutron source.

In the Section 3 is defined the integral quantity from which the importance function is obtained. In the Section 2, it presented the equations of point kinetics obtained from the space kinetics equations, with the use of a generic importance function, $\Psi^*(\vec{r},E,\widehat{\Omega})$. In Section 4, the kinetic parameters Λ, β, Γ and q are calculated and analyzed, for five different values of k_{eff}: 0.950, 0.960, 0.970, 0.980 and 0.990 and an external source considered constant, i.e., $s(\vec{r},E,\widehat{\Omega},t)=s_0(\vec{r},E,\widehat{\Omega})$. These parameters were also calculated and analyzed using the importance functions proposed by Gandini and Salvatores (2002), Dulla et al. (2006), Nishihara et al. (2003).

2. Point kinetics equations for ads

To obtain the point kinetics equations it is necessary to admit that from a certain instant on a transient occurs in the system, so that the appropriate equations that describe their behavior are the space kinetics equations. Thus for a subcritical system we can write

$$\frac{1}{v(E)}\frac{\partial}{\partial t}\varphi(\vec{r},E,\widehat{\Omega},t)+L\varphi(\vec{r},E,\widehat{\Omega},t) = F\varphi(\vec{r},E,\widehat{\Omega},t)-\sum_{i=1}^{6}F_i\varphi(\vec{r},E,\widehat{\Omega},t)+\frac{1}{4\pi}\sum_{i=1}^{6}\lambda_i\chi_i(E)C_i(\vec{r},t) + s(\vec{r},E,\widehat{\Omega},t) \tag{1}$$

and

$$\frac{1}{4\pi}\chi_i(E)\frac{\partial}{\partial t}C_i(\vec{r},t)=F_i\varphi(\vec{r},E,\widehat{\Omega},t)-\frac{1}{4\pi}\lambda_i\chi_i(E)C_i(\vec{r},t); \quad (i=1,2,\ldots,6). \tag{2}$$

where the transport (L) and production of neutrons by fission (F) operators are defined as follows:

$$L\equiv\widehat{\Omega}\cdot\vec{\nabla}(\circ)+\Sigma_t(\vec{r},E,t)(\circ)-\int_{4\pi}\int_0^\infty\Sigma_s(\vec{r},E'\to E,\widehat{\Omega}'\to\Omega,t)(\circ)dE'd\widehat{\Omega}'$$

and

$$F=F_p+F_i.$$

The operators for production due to prompt fission (F_p) and delayed neutrons (F_i) are given by

$$F_p\equiv\frac{1}{4\pi}\int_{4\pi}\int_0^\infty(1-\beta)\chi_p(E)v(E')\Sigma_f(\vec{r},E',t)(\circ)dE'd\widehat{\Omega}'$$

and

$$F_i\equiv\frac{1}{4\pi}\beta_i\int_{4\pi}\int_0^\infty\chi_i(E)v(E')\Sigma_f(\vec{r},E',t)(\circ)dE'd\widehat{\Omega}'.$$

Now, multiplying Eq. (1) by $\Psi^*(\vec{r},E,\widehat{\Omega})$ and integrating in the phase space, we obtain

$$\begin{aligned}&\frac{d}{dt}\int_V\int_{4\pi}\int_0^\infty\Psi^*(\vec{r},E,\widehat{\Omega})\frac{1}{v(E)}\varphi(\vec{r},E,\widehat{\Omega},t)dEd\widehat{\Omega}d^3r\\&\quad+\int_V\int_{4\pi}\int_0^\infty\Psi^*(\vec{r},E,\widehat{\Omega})L\varphi(\vec{r},E,\widehat{\Omega},t)dEd\widehat{\Omega}d^3r\\&=\int_V\int_{4\pi}\int_0^\infty\Psi^*(\vec{r},E,\widehat{\Omega})F\varphi(\vec{r},E,\widehat{\Omega},t)dEd\widehat{\Omega}d^3r\\&\quad-\sum_{i=1}^{6}\int_V\int_{4\pi}\int_0^\infty\Psi^*(\vec{r},E,\widehat{\Omega})F_i\varphi(\vec{r},E,\widehat{\Omega},t)dEd\widehat{\Omega}d^3r\\&\quad+\frac{1}{4\pi}\sum_{i=1}^{6}\lambda_i\int_V\int_{4\pi}\int_0^\infty\Psi^*(\vec{r},E,\widehat{\Omega})\chi_i(E)C_i(\vec{r},t)dEd\widehat{\Omega}d^3r\\&\quad+\int_V\int_{4\pi}\int_0^\infty\Psi^*(\vec{r},E,\widehat{\Omega})s(\vec{r},E,\vec{\Omega},t)dEd\widehat{\Omega}d^3r,\end{aligned}\tag{3}$$

where the importance function is the solution of the following equation:

$$L_0^+\Psi^*(\vec{r},E,\widehat{\Omega})=F_0^+\Psi^*(\vec{r},E,\widehat{\Omega})+s^+(\vec{r},E,\widehat{\Omega}) \tag{4}$$

and the term $s^+(\vec{r},E,\widehat{\Omega})$ is related to an integral quantity given by

$$Q=\int_V\int_{4\pi}\int_0^\infty s^+(\vec{r},E,\widehat{\Omega})\varphi_0(\vec{r},E,\widehat{\Omega})dEd\widehat{\Omega}d^3r, \tag{5}$$

with the angular neutron flux ($\varphi_0(\vec{r},E,\widehat{\Omega})$) as a solution of the following equation, for the steady state system:

$$L_0\varphi_0(\vec{r},E,\widehat{\Omega})=F_0\varphi_0(\vec{r},E,\widehat{\Omega})+s(\vec{r},E,\widehat{\Omega},t_0). \tag{6}$$

Now, multiplying Eq. (4) by $\varphi(\vec{r},E,\widehat{\Omega},t)$ and integrating in the phase space we have

$$\begin{aligned}&\int_V\int_{4\pi}\int_0^\infty\varphi(\vec{r},E,\widehat{\Omega},t)L_0^+\Psi^*(\vec{r},E,\widehat{\Omega})dEd\widehat{\Omega}d^3r\\&=\int_V\int_{4\pi}\int_0^\infty\varphi(\vec{r},E,\widehat{\Omega},t)F_0^+\Psi^*(\vec{r},E,\widehat{\Omega})dEd\widehat{\Omega}d^3r\\&\quad+\int_V\int_{4\pi}\int_0^\infty\varphi(\vec{r},E,\widehat{\Omega},t)s^+(\vec{r},E,\widehat{\Omega})dEd\widehat{\Omega}d^3r.\end{aligned}\tag{7}$$

But, from the definition of adjoint operator (Marchuk et al., 1996), Eq. (7) can be rewritten as

$$\begin{aligned}&\int_V\int_{4\pi}\int_0^\infty\Psi^*(\vec{r},E,\widehat{\Omega})L_0\varphi(\vec{r},E,\widehat{\Omega},t)dEd\widehat{\Omega}d^3r\\&=\int_V\int_{4\pi}\int_0^\infty\Psi^*(\vec{r},E,\widehat{\Omega})F_0\varphi(\vec{r},E,\widehat{\Omega},t)dEd\widehat{\Omega}d^3r\\&\quad+\int_V\int_{4\pi}\int_0^\infty s^+(\vec{r},E,\widehat{\Omega})\varphi(\vec{r},E,\widehat{\Omega},t)dEd\widehat{\Omega}d^3r.\end{aligned}\tag{8}$$

Subtracting Eq. (8) from (3), we have

$$\begin{aligned}&\frac{d}{dt}\int_V\int_{4\pi}\int_0^\infty\Psi^*(\vec{r},E,\widehat{\Omega})\frac{1}{v(E)}\varphi(\vec{r},E,\widehat{\Omega},t)dEd\widehat{\Omega}d^3r\\&=\int_V\int_{4\pi}\int_0^\infty\Psi^*(\vec{r},E,\widehat{\Omega})[\delta F-\delta L]\varphi(\vec{r},E,\widehat{\Omega},t)dEd\widehat{\Omega}d^3r\\&\quad+\frac{1}{4\pi}\sum_{i=1}^{6}\lambda_i\int_V\int_{4\pi}\int_0^\infty\Psi^*(\vec{r},E,\widehat{\Omega})\chi_i(E)C_i(\vec{r},t)dEd\widehat{\Omega}d^3r\\&\quad-\sum_{i=1}^{6}\int_V\int_{4\pi}\int_0^\infty\Psi^*(\vec{r},E,\widehat{\Omega})F_i\varphi(\vec{r},E,\widehat{\Omega},t)dEd\widehat{\Omega}d^3r\\&\quad+\int_V\int_{4\pi}\int_0^\infty\Psi^*(\vec{r},E,\widehat{\Omega})s(\vec{r},E,\vec{\Omega},t)dEd\widehat{\Omega}d^3r\\&\quad-\int_V\int_{4\pi}\int_0^\infty s^+(\vec{r},E,\widehat{\Omega})\varphi(\vec{r},E,\widehat{\Omega},t)dEd\widehat{\Omega}d^3r,\end{aligned}\tag{9}$$

where

$$\delta L\equiv L-L_0=\delta\Sigma_t(\vec{r},E,t)(\circ)-\int_{4\pi}\int_0^\infty\delta\Sigma_s(\vec{r},E'\to E,\widehat{\Omega}'\to\Omega,t)(\circ)dE'd\widehat{\Omega}'$$

and

$$\delta F\equiv F-F_0=\frac{1}{4\pi}\int_{4\pi}\int_0^\infty\{(1-\beta)\chi_p(E)+\sum_{i=1}^{6}\beta_i\chi_i(E)\}v(E')\delta\Sigma_f(\vec{r},E',t)(\circ)dE'd\widehat{\Omega}',$$

with

$$\delta\Sigma_t(\vec{r},E,t)\equiv\Sigma_t(\vec{r},E,t)-\Sigma_t(\vec{r},E,t_0),$$
$$\delta\Sigma_f(\vec{r},E,t)\equiv\Sigma_f(\vec{r},E,t)-\Sigma_f(\vec{r},E,t_0)$$

and

$$\delta\Sigma_s(\vec{r},E'\to E,\widehat{\Omega}'\to\Omega,t)=\Sigma_s(\vec{r},E'\to E,\widehat{\Omega}'\to\Omega,t)-\Sigma_s(\vec{r},E'\to E,\widehat{\Omega}'\to\Omega,t_0).$$