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Eigenvalue implicit sensitivity and uncertainty analysis with the subgroup resonance-calculation method

Yong Liu [a], Liangzhi Cao [a], Hongchun Wu [a], Tiejun Zu [a,*], Wei Shen [a,b]

[a] Xi'an Jiaotong University, Xi'an, Shaanxi, China
[b] Canadian Nuclear Safety Commission, Ottawa, Ontario, Canada

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ABSTRACT

Response sensitivity coefficients with respect to nuclide cross sections consist of two parts, explicit sensitivity coefficients and implicit sensitivity coefficients. The explicit sensitivity coefficients, which account the direct impact of cross sections on the responses through neutron transport equation, can be calculated efficiently with the classical Perturbation Theory. The implicit sensitivity coefficients, which account the indirect impact of cross sections on the responses through resonance self-shielding, are either omitted in most sensitivity analysis codes, or accounted for based on simple resonance-calculation methods which are not applicable for complex fuel designs. In order to expand the implicit sensitivity analysis method to wider application domain, a method based on the Generalized Perturbation Theory (GPT) is proposed in this paper to calculate the implicit sensitivity coefficients by using the subgroup method in the resonance self-shielding calculation. Based on the in-house-developed 2-D general-geometry method-of-characteristic neutron-transport code AutoMOC and subgroup resonance self-shielding code SUGAR, the proposed method has been implemented in the COLEUS code for the sensitivity and uncertainty analysis. Numerical analysis is then performed to investigate the impact of the implicit sensitivity coefficients of eigenvalue on non-resonance nuclide cross sections in two single-cell cases with different enrichments. The eigenvalue sensitivity coefficients predicted by the COLEUS code are consistent with those calculated by the direct-perturbation method, the reference solution. The results show that the implicit sensitivity has an important impact on both sensitivity and uncertainty in some analyzed cases.

1. Introduction

The results of reactor physics calculation are used for reactor design, predicting the properties and behavior under various operating conditions. The accuracy of the results affects the margin of design, limitation of controlling, and even the economy and safety of the reactor. However, because of the hypotheses, approximations in the solution model and the inaccuracy of the input parameters, the results lie in a certain range, which mean that the results are uncertain. Traditionally, conservative assumptions and large safety margins are used in reactor safety analysis because the size of uncertainty is not quantified. In this situation, the safety may be guaranteed with the sacrifice of economy. For example, the uncertainty of neutron fluence in the pressure vessel will affect the choice of safety margins, and consequently affect the operating conditions, the life of nuclear installations, and the cost (Kodeli, 2001). However, if the size of the uncertainty can be determined, the design and operation will be more valid. As a result, the economy can be realized on the premise of the assurance of safety.

Neutron-transport calculation is the first calculation step that will introduce uncertainties. The uncertainties will be propagated through this step to the subsequent calculation steps. Therefore, neutron-transport calculation uncertainty analysis is one of the basic uncertainty analysis in reactor calculation. It is generally believed that the uncertainties of calculated responses in neutron-transport calculation stem from three sources (Weisbin et al., 1976; Laletin and Kovalishin, 2002): (1) modeling error, which is related to inaccuracy of mathematic-physical model, (2) numerical error, which is related to inaccuracy numerical methods, (3) input-parameter error including error in the nuclear data library. Uncertainty introduced by the nuclear data is considered as one of the most significant uncertainty in neutron-transport calculation (Pusa, 2012). As a consequence, the research of response sensitivity and uncertainty with respect to nuclide cross sections obtains more and more attention.

* Corresponding author at: No. 28 Xianning West Road, Xi'an 710049, China. Tel.: +86 029 82663285; fax: +86 029 82667802.
E-mail address: tiejun@mail.xjtu.edu.cn (T. Zu).

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Sensitivity calculation is a necessary calculation step in deterministic-method-based uncertainty analysis. Response (eigenvalue, reaction rate, etc.) sensitivity with respect to cross sections can be divided into two parts, namely explicit sensitivity and implicit sensitivity. The former is the direct impact of cross sections perturbation on the responses through neutron transport equation, while the latter is the indirect impact of cross sections perturbation on the responses through resonance self-shielding procedure (Williams et al., 2001). As an indirect impact related with resonance calculation, implicit sensitivity is often neglected in many sensitivity and uncertainty analysis, and many sensitivity and uncertainty analysis codes lack the ability to perform implicit sensitivity calculation. However, from the original research of Greenspan et al. (1978) to the subsequent research of Williams et al. (2001), the results indicated that the implicit sensitivity had a non-negligible importance relative to the explicit sensitivity and the implicit effect had a magnitude that was more that 40% of the explicit effect in some cases.

Therefore, it is necessary and important to take implicit sensitivity into account when sensitivity and uncertainty analysis is performed. Up to now, however, most implicit sensitivity studies are mainly established for simple resonance-calculation methods such as Bondarenko method (Williams et al., 2001), generalized Stamm'ler method (Dion and Marleau, 2013), and so on (Kimura and Kitada, 2012), which are not applicable for complex fuel and core designs. Moreover, the impact of the implicit sensitivity on the uncertainty results was not publically reported before. Recently, the subgroup resonance-calculation method (Hébert, 2007) and the method-of-characteristics (MOC) transport-calculation method (Askew, 1972) have been widely used for complex geometry fuel and core designs and show high adaptability. In order to extend the implicit sensitivity analysis method to a wider application extent and investigate the impact of the implicit sensitivity on the uncertainty results, the eigenvalue implicit sensitivity and uncertainty analysis with the subgroup resonance-calculation method are carried out in this paper. The sensitivity-calculation methods are based on the classical Perturbation Theory (PT) for the explicit sensitivity and the Generalized Perturbation Theory (GPT) for the implicit sensitivity. And then the eigenvalue uncertainty is determined based on the Sandwich rule (Alfassi, 2004) and the covariance library is created based on ENDF/B-VII.1 using NJOY (MacFarlane and Muir, 1994).

This paper is organized as follows: Theoretical models of this work are described in Section 2. Section 3 describes the implementation of the calculation procedure based on the method presented in Section 2. Section 4 gives the calculation results of two representative fuel-pin cells with different enrichments. Finally, Section 5 summarizes and concludes the work.

2. Theoretical models

2.1. Sensitivity coefficient

In this paper, the mathematical model is multigroup neutron transport equation that can be written in operator form as:

$$(\boldsymbol{L}-\lambda\boldsymbol{F})\Phi=0 \tag{1}$$

where $\boldsymbol{L}$ is multigroup form of the Boltzmann loss operator for neutrons; $\boldsymbol{F}$ is multigroup form of the Boltzmann production operator for neutrons; $\lambda = 1/k_{\text{eff}}$ is the minimum eigenvalue for this equation; $\Phi = \Phi(\boldsymbol{r}, \boldsymbol{\Omega}, g)$ is multigroup neutron angular flux; and $\boldsymbol{r}$ is position, $\boldsymbol{\Omega}$ is direction of travel, g is energy group; k_{eff} is effective multiplication factor.

When a parameter perturbation is small enough, the change of k_{eff} can be expressed through the linear relationship:

$$\frac{\Delta k_{\text{eff}}}{k}=S^{tot}_{k_{\text{eff}},\alpha}\frac{\Delta\alpha}{\alpha} \tag{2}$$

where α stands for a multigroup microscopic cross section of a particular nuclide; $S^{tot}_{k_{\text{eff}},\alpha}$ stands for the total sensitivity coefficient that indicates the relative percent change of k_{eff} when α is perturbed with a relative value of 1%.

If the perturbation of α just affects the corresponding multigroup macro cross section, and then affects the eigenvalue through transport calculation, the sensitivity coefficient can be considered as explicit sensitivity coefficient. If the perturbation of α affects the self-shielding cross sections and then affects the multigroup macro cross section, and then affects the eigenvalue through transport calculation, the sensitivity coefficient can be considered as implicit sensitivity coefficient. So the total sensitivity coefficient of eigenvalue can be expressed as:

$$\begin{aligned}S^{\text{tot}}_{k_{\text{eff}},\alpha}&=\frac{\alpha}{k_{\text{eff}}}\frac{\mathrm{d}k_{\text{eff}}}{\mathrm{d}\alpha}=\frac{\alpha}{k_{\text{eff}}}\frac{\partial k_{\text{eff}}}{\partial\alpha}+\frac{\alpha}{k_{\text{eff}}}\sum_{j,x,g}\frac{\partial k_{\text{eff}}}{\partial\sigma^{(j)}_{x,g}}\frac{\partial\sigma^{(j)}_{x,g}}{\partial\alpha}\\&=S^{\text{exp}}_{k_{\text{eff}},\alpha}+S^{\text{imp}}_{k_{\text{eff}},\alpha}\end{aligned} \tag{3}$$

$S^{\text{exp}}_{k_{\text{eff}},\alpha}$ is explicit sensitivity coefficient and $S^{\text{imp}}_{k_{\text{eff}},\alpha}$ is implicit sensitivity coefficient. $\sigma^{(j)}_{x,g}$ is the gth group effective self-shielding cross section of reaction type x for nuclide j.

2.1.1. Explicit sensitivity

If α is explicitly included in the multigroup Boltzmann transport operator, the explicit sensitivity coefficient of k_{eff} with respect to α is defined as:

$$S^{\text{exp}}_{k_{\text{eff}},\alpha}=\frac{\alpha}{k_{\text{eff}}}\frac{\partial k_{\text{eff}}}{\partial\alpha} \tag{4}$$

The explicit sensitivity coefficient can be efficiently calculated utilizing classical Perturbation Theory and it equals to Weisbin et al. (1976):

$$S^{\text{exp}}_{k_{\text{eff}},\alpha}=\frac{\alpha}{k_{\text{eff}}}\frac{\partial k_{\text{eff}}}{\partial\alpha}=-\frac{\alpha}{k_{\text{eff}}}\frac{\left\langle\Phi^{*},\left(\frac{\partial L}{\partial\alpha}-\frac{1}{k_{\text{eff}}}\frac{\partial F}{\partial\alpha}\right)\Phi\right\rangle}{\left\langle\Phi^{*},\frac{1}{k^{2}_{\text{eff}}}F\Phi\right\rangle} \tag{5}$$

where $\langle,\rangle$ indicates integration over space, direction, energy group. Φ^* is adjoint flux, which is the solution of the corresponding adjoint multigroup transport equation:

$$(\boldsymbol{L}^{*}-\lambda\boldsymbol{F}^{*})\Phi^{*}=0 \tag{6}$$

where $\boldsymbol{L}^*$ and $\boldsymbol{F}^*$ is the adjoint operators of $\boldsymbol{L}$ and $\boldsymbol{F}$, respectively.

From Eq. (5), it can be found that the explicit sensitivity coefficients of k_{eff} with respect to all parameters directly appearing in the transport operator can be efficiently obtained with only once forward calculation and once adjoint calculation. In this paper, a subgroup method based code SUGAR (Cao et al., 2011 and He et al., 2014) is applied to the resonance calculation, and a transport code AutoMOC (Chen et al., 2010) based on two-dimensional arbitrary-geometry method-of-characteristics is applied to solve the forward and adjoint neutron transport equation calculation.

2.1.2. Implicit sensitivity

To simplify notation, let α stand for the multigroup cross section of a non-resonance nuclide, and $\sigma^{j}_{r,x,g}$ stand for the self-shielding multigroup cross section of isotope j in region r for reaction type x and energy group g, and $\phi_r(E)$ stand for neutron scalar flux in region r. The symbols j and r are omitted in the following equations for simplicity. Generally, $\sigma_{x,g}$ is given by:

$$\sigma_{x,g}=\frac{\int_{g}\sigma_{x}(E)\phi(E)dE}{\int_{g}\phi(E)dE} \tag{7}$$