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Uncertainty analysis using Beta-Bayesian approach in nuclear safety code validation

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ABSTRACT

Since best-estimate plus uncertainty analysis was approved by Nuclear Regulatory Commission for nuclear reactor safety evaluation, several uncertainty assessment methods have been proposed and applied in the framework of best-estimate code validation in nuclear industry. Among them, the Wilks' method and Bayesian approach are the two most popular statistical methods for uncertainty quantification. This study explores the inherent relation between the two methods using the Beta distribution function as the prior in the Bayesian analysis. Subsequently, the Wilks' method can be considered as a special case of Beta-Bayesian approach, equivalent to the conservative case with Wallis' "pessimistic" prior in the Bayesian analysis. However, the results do depend on the choice of the pessimistic prior function forms. The analysis of mean and variance through Beta-Bayesian approach provides insight into the Wilks' 95/95 results with different orders. It indicates that the 95/95 results of Wilks' method become more accurate and more precise with the increasing of the order. Furthermore, Bayesian updating process is well demonstrated in the code validation practice. The selection of updating prior can make use of the current experience of the code failure and success statistics, so as to effectively predict further needed number of numerical simulations to reach the 95/95 criterion.

1. Introduction

In the 1988 amendment of the 10 CFR50.46, best-estimate plus uncertainty (BEPU) analysis was approved by Nuclear Regulatory Commission (NRC) as an alternative of the conservative Appendix K approach for nuclear safety analysis (USNRC, 1988). The computational uncertainty is required to be quantified to satisfy the "95/95 criterion", i.e., the parameters of interest with 95% probability is below the prescribed limit (e.g. PCT 2200 °F) with 95% confidence. The Code Scaling, Applicability and Uncertainty (CSAU) methodology was then developed by NRC as a basic framework of uncertainty analysis (Technical Program Group, 1989), in which response surface was applied for parametric treatment of uncertainties. Following CSAU methodology, the demanding calculation cost of the response surface method significantly affects its actual application in the industry. Thereafter, several statistical techniques have been proposed to improve the effectiveness of uncertainty quantification. Among them, the Wilks' method and the Bayesian approach are widely accepted to quantify the uncertainty for code validation.

According to Wilks' statistic theory (Wilks, 1941), non-parametric order statistics can be used to determine the tolerance limits for random samples. In 1985, GRS employed the Wilks' method and demonstrated its application to determine the needed number of code runs in nuclear safety analysis (Glaeser et al., 1994). The main idea of Wilks' method is to establish a tolerance interval (L, U) that contains at least a fraction (γ) of the population with a given confidence level (β), which can be mathematically expressed by

$$P\left(\int_L^U f(x)dx \geq \gamma\right) = \beta \tag{1}$$

Here, x is an arbitrary variable with the probability density function $f(x)$, P denotes the probability. If the interval lies below the safety limit value, we declare the operation safe. Given the required value of β and γ, it becomes possible to determine the needed number of code runs. To meet the 95/95 criterion (i.e. $\beta = \gamma = 0.95$), only 59 continuously successful code runs are required and the maximum output of the parameter of interest is considered as the upper limit against specific safety limit. Wilks' method is simple and efficient for the reduction of sample sizes, independent of the number of

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input uncertain parameters. Thus, Wilks' method is widely used in uncertainty analysis.

Bayesian approach is also a popular uncertainty assessment method. It is based on the framework of subjective probability, i.e. a subjective degree of belief, as a counterpart of traditional 'frequentist' definition of probability (Siu and Kelly, 1998). Bayesian technique is able to incorporate a wide variety of information types, e.g. experience judgment as well as existing statistical data, in the estimation process. Bayesian approach provides the mathematical means of combining the prior state of knowledge and the data evidence to calculate the posterior estimation in an effective way. For its application in code validation, Bayesian approach is given by the following equation:

$$\pi(p/E) = \frac{\pi_0(p)L(E/p)}{\int_0^1 L(E/P)\pi_0(p)dp} \tag{2}$$

where $\pi_0(p)$ is the prior probability distribution of the probability p; $L(E/p)$ denotes the likelihood of the evidence E, given p; and $\pi(p/E)$ represents the posterior probability distribution of p under the evidence E.

According to Wallis (2007), a "pessimistic" prior distribution was employed in the Bayesian approach, leading to the same number of code runs required by the Wilks' method, and suggesting that the choice of prior probability makes little difference as long as it does not contain prior failures. In this study, however, a general Beta distribution function is chosen as the prior, for the purpose of exploring the relation between the two methods. This analysis indicates that Wallis' "pessimistic" prior is a special case of a Beta distribution that results in a conclusion identical to Wilks' method. The prior selection does affect the required number of code runs to reach the 95/95 criterion, i.e., a more "pessimistic" prior than Wallis' selection could result in a greater number of required code runs. Furthermore, discussions are provided for the application of general Beta-Bayesian approach in code validation. The accuracy and precision of the Wilks' 95/95 results with different orders are well estimated from the perspective of the Beta-Bayesian approach. Also, the Bayesian updating process is well demonstrated in code validation practice to predict further required code runs on the basis of the current calculation experience.

2. The relation between Wilks' method and Bayesian approach

From the derivation of the well-known Wilks' formula (Pal and Makai, 2002), it implies that the Probability Density Function (PDF) of a success probability for code runs against specific safety limit is essentially a kind of Beta distribution. Meanwhile, Beta distribution could be chosen as a general prior prediction in the Bayesian approach since it can take a wide variety of different shapes that can simulate various forms of prior experience. In order to explore the relation between these two methods, the characteristic of Beta distribution will be described here.

The Beta distribution is a family of continuous probability distributions defined on the interval [0,1], characterized by two positive shape parameters, α and β. The fundamental properties of the Beta distribution function are listed as follows:

Probability Density Function (PDF):

$$f(x;\alpha,\beta) = \frac{x^{\alpha-1}(1-x)^{\beta-1}}{\int_0^1 u^{\alpha-1}(1-u)^{\beta-1}du} = \frac{x^{\alpha-1}(1-x)^{\beta-1}}{B(\alpha,\beta)}$$

Cumulative Distribution Function (CDF):

$$F(x;\alpha,\beta) = \frac{\int_0^x u^{\alpha-1}(1-u)^{\beta-1}du}{\int_0^1 u^{\alpha-1}(1-u)^{\beta-1}du} = I_x(\alpha,\beta)$$

where $B(\alpha,\beta)$ is complete beta function; $I_x(\alpha,\beta)$ is regularized incomplete beta function.

Mean:

$$\mu = E(X) = \int_0^1 xf(x;\alpha,\beta)dx = \frac{\alpha}{\alpha+\beta}$$

Variance:

$$Var(X) = E[(X-\mu)^2] = \frac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)}$$

Beta distribution presents symmetric shapes when $\alpha = \beta$ as shown in Fig. 1, and asymmetric shapes when $\alpha \neq \beta$ as in Fig. 2. It embodies a series of distribution shapes, like uniform, U-shaped, unimodal-shaped, J-shaped, triangular distributions and so on, depending on the values of α and β. Moreover, the distribution skews towards right side when α is relatively greater than β, whereas it skews towards left side when α is smaller than β.

In the Bayesian approach, "conjugate pair" is a set of special combination to result in the posterior with the same functional form as the prior. The combination of Beta prior and binomial likelihood is a classic "conjugate pair" commonly used in the application of Bayesian approach. Generally, the choice of a prior can be in any form of distributions. However, Beta distribution is proved to be a rational choice of prior for the application in code validation based on the following reasons. First, the characteristic of Beta distribution satisfies the requirement of probability. Because the range of the variable is [0, 1], and the shape parameters, α and β, appear respectively on the exponent of p and $(1-p)$, it should be

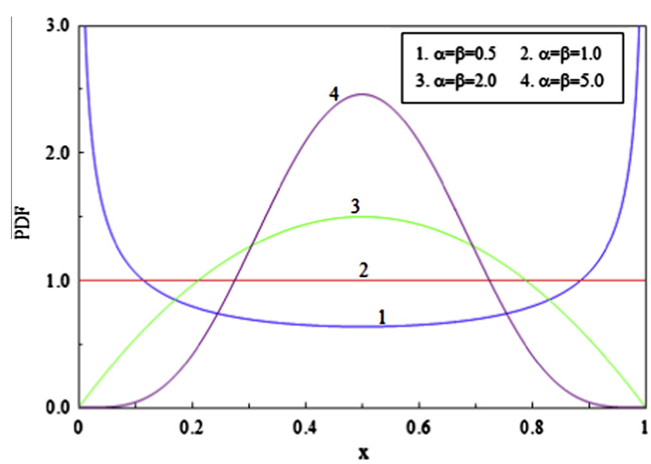

Fig. 1. Symmetric Beta distributions with $\alpha = \beta$.

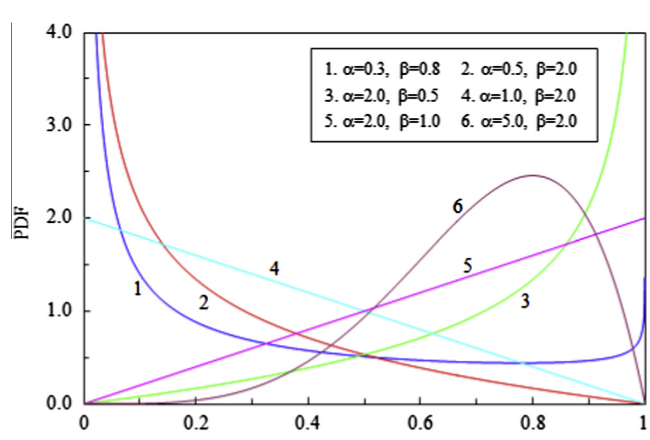

Fig. 2. Asymmetric Beta distributions with $\alpha \neq \beta$.