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Bond graph representation of nuclear reactor point kinetics and nearly incompressible thermal hydraulics

Eugeny Sosnovsky *, Benoit Forget *

Computational Reactor Physics Group, Department of Nuclear Science and Engineering, Massachusetts Institute of Technology, 77 Massachusetts Avenue, Cambridge, MA 02139, United States

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ABSTRACT

This work presents a simplified 1D model for a pressurized water reactor core, suitable for very rapid transients like control rod ejection. The model is represented using the bond graph formalism, a technique for modeling engineering systems as combinations of connected elements. Bond graphs are a flexible way of presenting coupled physics problems by automating the computer science aspects of modeling and letting the modelers focus on the physics; they were introduced in earlier work.

To help leverage the flexibility of bond graph representations of physical systems, a new bond graph processing code, BGSolver, is introduced. BGSolver has been developed by the authors over the past several years, and is now released as open source software.

A rapid rod ejection benchmark is solved with both BGSolver and RELAP5-3D; BGSolver obtained full convergence with a 5 ms time step, while RELAP5-3D required a 1 ms time step, due to the fully coupled time integration that BGSolver employed, compared to an operator splitting-based time integrator of RELAP5-3D. BGSolver's time integrator demonstrated 3rd-order convergence in time, a very desirable property. A single nonlinear solve was used to obtain the steady state with BGSolver.

1. Introduction

In nuclear reactor transients, the two most important physics are neutron transport and thermal hydraulics. These physics are coupled: fission creates a significant heat source in a reactor, while temperature and density variations (particularly moderator density) affect reactor reactivity.

Depending on the desired fidelity, transient neutron transport can either be treated as a full spatially distributed problem (spatial neutron kinetics), or as a point kinetics problem, in which only the amplitude of the fission power source in the reactor varies. The assumption of a steady shape is the limiting one in such models. They are rarely used for detailed safety analysis, however, they are much simpler to construct and solve, and so are frequently used for:

1. Operator training and examination: the point kinetics model is sufficiently small to be solved in real time, which allows it to be used together with systems codes in training scenarios.
2. Coarse systems-level thermal hydraulic models for safety transient analysis, particularly the ones done by a single thermal hydraulic code package; such codes typically do not have full flexible spatial kinetics capabilities. One notable example is RELAP5-3D, which incorporates a full point kinetics representation (The RELAP5-3D Code Development Team, 2005).
3. Control system design and optimization.

Whether or not point or spatial kinetics are used, thermal hydraulics and neutron transport coupling is a complicated multiphysics problem. Such problems are normally treated through operator splitting, but recently, there has been a push to use fully coupled simulation instead (Knoll and Keyes, 2004; Gaston et al., 2009). One of the approaches used to develop non-physics-specific fully coupled code packages is the bond graph formalism: a set of techniques for representing the problems physics in terms of a connected system of graph elements, which is then automatically converted into a state derivative vector and integrated. The bond graph formalism itself, the motivations for its use, as well as its application to spatial neutron kinetics with nonlinear thermal feedback, are described in detail in Sosnovsky and Forget (2013).

One of the earliest applications of bond graphs to reactor analysis was the bond graph representation of a linear point kinetics model with a lumped parameter thermal feedback (Tylee, 1981, 1986). At the time, bond graph processing codes could only process fully linear, constant coefficient bond graph models, which lead to a linearized model being used. The code ENPORT (Rosenberg, 1985) was used in Tylee (1981). This model has significant inaccuracies for a

* Address: Massachusetts Institute of Technology, 77 Massachusetts Avenue, Room 24-214, Cambridge, MA 02139, United States. Tel.: +1 603 380 1847 (E. Sosnovsky). Tel.: +1 617 253 1655 (B. Forget).

E-mail addresses: esos@mit.edu (E. Sosnovsky), bforget@mit.edu (B. Forget).

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number of transients, and so, if a bond graph-based model is to be used for any of the above applications today, it is necessary to:

1. Convert the bond graph representation of a linearized point kinetics model with a lumped parameter thermal feedback to a nonlinear point kinetics model with a spatially distributed feedback.
2. Develop a bond graph processing code capable of handling nonlinear bond graph models.

The present paper accomplishes these two objectives. A nonlinear point kinetics model with a 1D nearly incompressible flow feedback is summarized in Section 2; its bond graph representation is given in Section 3. A nonlinear bond graph processing code `BGSolver`, previously used in Sosnovsky and Forget (2013), and recently released as open source software, is presented in Section 4. A benchmark problem, solved both using the bond graph representation with `BGSolver`, and RELAP5-3D, is given in Section 5. Section 6 details the paper's conclusions and future work.

2. Nonlinear point kinetics model with spatially distributed feedback

A nonlinear point kinetics model of a reactor with a simplified spatially distributed feedback consists of two parts: the point kinetics equations system, and the thermal hydraulic equations. These parts are discussed in Sections 2.1 and 2.2, respectively.

2.1. Nonlinear point kinetics equations

Assuming a constant spatial shape of the flux, a constant spectrum and constant delayed neutron fractions, the point kinetics of a system with thermal feedback are represented by the following equations:

$$\frac{d}{dt}N(t) = \left(\frac{\rho(t)-\beta}{\Lambda}\right)N(t) + \sum_{m=1}^{M}\lambda_m C_m(t), \tag{1a}$$

$$\frac{d}{dt}C_m(t) = \left(\frac{\beta_m}{\Lambda}\right)N(t) - \lambda_m C_m(t) \quad \forall m \in [1,\ldots,M], \tag{1b}$$

in which $N(t)$ is the number of neutrons in the core at time t, $\rho(t)$, β and Λ are the reactor reactivity, delayed neutron fraction and prompt neutron generation time, respectively, m is the precursor family index (1 to M), and β_m, λ_m and $C_m(t)$ are the precursor family m delayed neutron fraction, decay constant and number of precursor nuclei in the core, respectively.

Under the above assumptions, $P(t)$, the power generated in the core, is linearly proportional to the number of neutrons in the core through a constant K. Similarly, the precursor family m delayed neutron power $\widetilde{C}_m(t)$ (a derived quantity), is also linearly proportional to $C_m(t)$, through the same constant:

$$P(t) = KN(t), \tag{2a}$$

$$\widetilde{C}_m(t) = KC_m(t). \tag{2b}$$

Multiplying Eq. (1) by K yields:

$$\frac{d}{dt}P(t) = \left(\frac{\rho(t)-\beta}{\Lambda}\right)P(t) + \sum_{m=1}^{M}\lambda_m \widetilde{C}_m(t), \tag{3a}$$

$$\frac{d}{dt}\widetilde{C}_m(t) = \left(\frac{\beta_m}{\Lambda}\right)P(t) - \lambda_m \widetilde{C}_m(t) \quad \forall m \in [1,\ldots,M]. \tag{3b}$$

For the purposes of bond graph representation (Section 3), it is convenient to rewrite Eq. (3b) in terms of $\widetilde{C}^*_m(t)$:

$$\widetilde{C}^*_m(t) = \frac{\Lambda}{\beta_m}\widetilde{C}_m(t). \tag{4}$$

Substituting Eq. (4) into Eq. (3) yields:

$$\frac{d}{dt}P(t) = \left(\frac{\rho(t)-\beta}{\Lambda}\right)P(t) + \sum_{m=1}^{M}\frac{\lambda_m\beta_m}{\Lambda}\widetilde{C}^*_m(t), \tag{5a}$$

$$\frac{d}{dt}\widetilde{C}^*_m(t) = P(t) - \lambda_m\widetilde{C}^*_m(t). \tag{5b}$$

Feedback in point kinetics models varies in complexity, but generally consists of the reactor reactivity $\rho(t)$ being dependent on one or more of the following parameters:

- Fuel temperature (fuel Doppler feedback).
- Moderator temperature.
- Moderator density.
- Boron concentration.
- Structure temperature, structural motion (rarely modeled).

Regardless of the model of feedback, reactivity is given by:

$$\rho(t) = \rho_{ex}(t) - \rho_b + \rho_{fb}(t), \tag{6}$$

in which $\rho_{ex}(t)$ is the external reactivity (usually due to control rod/blade movement), ρ_b is the bias reactivity (used to enforce a desired initial reactivity $\rho^0 = \rho(t^0)$, which is zero for most transients), and $\rho_{fb}(t)$ is the reactivity feedback term. From here forward superscript 0 denotes initial/nominal quantities.

Point kinetics represent the neutron population in the reactor using a single scalar (a "lumped parameter" approach), but the associated thermal hydraulic model may be either lumped or spatially distributed. A lumped model does not divide the thermohydraulic compositions (fuel, moderator, etc.) into individual regions, as done in a spatially distributed model. Additionally, feedback models can be either separable, or coupled ("tabular," or "multidimensional"). In a separable model, the change in feedback reactivity due to a change in a region's temperature/density/boron concentration is independent of all other regions' thermohydraulic states. One can postulate a coupled model in which the entire thermohydraulic state vector of the system affects the feedback reactivity, with each region's state's individual contributions being dependent on the thermohydraulic state of the system. Such model would be prohibitively expensive to implement, and so instead, coupled models are implemented as weighted averages of each composition's thermohydraulic state variables affecting the feedback reactivity together, and the averages' individual contributions being dependent on the thermohydraulic state of the system. This is the model presented below.

The spatially distributed separable reactivity feedback model is given by Eq. (7). The spatially distributed coupled feedback model, as described above, is given by Eq. (8). With N_k being the number of regions in a composition k, the lumped models can be written similarly, with $N_k = 1$ for all k. Eqs. (7) and (8) represent only the thermal feedback, but additional terms can be naturally added to account for other feedback thermohydraulic variables, as listed above.

$$\rho_{fb}(t) = \sum_{k=1}^{K_c}\sum_{r=1}^{N_k}\alpha_{T,r,k}T_{r,k}(t) + \sum_{k=1}^{K_c}\sum_{r=1}^{N_k}W_{T,r,k}R_{T,r,k}(T_{r,k}(t)), \tag{7}$$

$$\rho_{fb}(t) = R\left(\sum_{r=1}^{N_1}W_{T,r,1}T_{r,1}(t),\ldots,\sum_{r=1}^{N_{K_c}}W_{T,r,K_c}T_{r,K_c}(t)\right). \tag{8}$$

Here, for region r of composition k, $T_{r,k}(t)$ is the temperature (if ρ_b is present) or its deviation from nominal, $\alpha_{T,r,k}$ is the thermal feedback coefficient, and $R_{T,r,k}(T_{r,k}(t))$ is the nonlinear thermal feedback function. In Eq. (7), $W_{T,r,k}$ is the nonlinear thermal feedback weighting factor, and in Eq. (8) it is the thermal feedback weighting factor to be used for the weighted average calculation