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A practical implementation of the higher-order transverse-integrated nodal diffusion method

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ABSTRACT

Transverse-integrated nodal diffusion methods currently represent the standard in full core neutronic simulation. The primary shortcoming of this approach is the utilization of the quadratic transverse leakage approximation. This approach, although proven to work well for typical LWR problems, is not consistent with the formulation of nodal methods and can cause accuracy and convergence problems. In this work, an improved, consistent quadratic leakage approximation is formulated, which derives from the class of higher-order nodal methods developed some years ago. Further, a number of iteration schemes are developed around this consistent quadratic leakage approximation which yields accurate node average results in much improved calculational times. The most promising of these iteration schemes results from utilizing the consistent leakage approximation as a correction method to the standard quadratic leakage approximation. Numerical results are demonstrated on a set of benchmark problems and further applied to a realistic reactor problem, particularly the SAFARI-1 reactor, operating at Necsa, South Africa. The final optimal solution strategy is packaged into a standalone module which may simply be coupled to existing nodal diffusion codes.

1. Introduction

The class of nodal diffusion methods, as applied to global reactor calculations (Duderstadt and Hamilton, 1976; Stamm'ler and Abbate, 1983), has grown into a mature and trusted technique in recent times. Nodal methods first appeared in reactor literature in the 1960s. This class of methods was originally developed in order to obtain, in a less rigorous, but more computationally efficient way, information about flux averages over fairly large spatially homogeneous regions or "nodes", from which the name "nodal methods" was derived. An important step in the development was when these methods became "consistent", in the sense that they limited to the fine-mesh finite-difference solution for decreasing mesh size, as described by Lawrence (1986). The motivation for these earlier nodal methods was to reduce the computational expense of traditional finite-difference methods for multi-group diffusion calculations over large cores requiring many mesh points. This motivation for the use of nodal methods continues up to the present, as do fundamental questions of homogenization techniques and flux reconstruction (dehomogenization) (Grossman and Hennart, 2007). In Dorning (1979) a subtle distinction is made between nodal and coarse-mesh methods, in claiming that true nodal methods, in contrast to many forms of coarse-mesh methods, do not yield the full intra-nodal flux shape as part of the solution, since only node-averaged quantities are available from the solution. This is an important distinction within the context of this work and we will adopt this differentiation.

In the 1970s the class of transversely-integrated nodal methods was developed in the form of the Nodal Expansion Method (NEM) (Finnemann et al., 1977), the use of integral nodal equations (Nodal Greens Function Method) (Lawrence and Dorning, 1980) and the Analytic Nodal Method (Smith, 1979). A mix of the NEM and ANM approaches was further developed some time later in the form of the AFEN code by Noh and Cho (1993). During the late 1980s a further development in nodal methods included the emergence of higher-order nodal methods (Ougouag and Rajić, 1988; Altiparmakov and Tomašević, 1990; Guessous and Akhmouch, 2002) which in principle could reproduce the full intra-nodal flux solution and steered away from the concept of simply requiring averages as primary unknowns. This development, although promising, incurred significant calculational cost and did not enter the mainstream of nodal diffusion codes. During the 1990s a number of developments enhanced the maturity of nodal methods and extended their shelf-life well into the present era. These include, amongst others, intra-nodal cross-sections shape feedback (Wagner et al., 1981), the generalization of the ANM to full multi-group (Vogel and weiss, 1992), the inclusion of non-linear extensions

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such as nodal rehomogenization (Smith, 1994), and more recently axial homogenization (Smith, 1992; Reitsma and Muller, 2002) techniques to eradicate the well known cusping effect. In 1989, as part of the NODEX code (Sutton, 1989), the Coarse Mesh Finite-Difference (CMFD) iteration scheme, described in Sutton and Aviles (1996), was developed. It has proven to be a successful acceleration approach and as such has been implemented in many modern nodal diffusion methods or codes. In this approach, the nodal equations are cast into a finite-difference form and nodal calculations are utilized to generate corrections to standard finite-difference node coupling coefficients in an iterative sense.

With regard to geometric flexibility, contributions in this regard would include the extension of the Analytic Nodal Method to hexagonal geometry (Chao, 1995), as well as to cylindrical geometry (Prinsloo and Tomašević, 2008). Both of these developments applied approaches of conformal mapping in order to preserve, as far as possible, the structure of equations from Cartesian geometry solvers.

A significant number of refinements in the application of nodal methods continue to be visible in literature, with the most attention being paid to improved intra-nodal depletion and flux reconstruction issues. Nevertheless, these methods have reached a level of maturity and it can be expected that developments in nodal diffusion methods will continue in an evolutionary manner until full core transport approaches become practical. An important bridging step in this regard is the so-called semi-heterogeneous methods which aim at limiting the impact of homogenization by subdividing assemblies into various spectral zones (Bahadir and Lindahl, 2009).

1.1. Transversely-integrated nodal methods and the transverse leakage approximation

Within the background described above, the class of transversely-integrated nodal methods is still quite prevalent in the industry today and addressing the issues pertaining to these code systems will still add value for a number of years to come. The class of transversely-integrated nodal methods provide substantial performance increases, while simultaneously maintaining high levels of accuracy. These methods mostly share three characteristics (Tomašević, 1996):

1. Unknowns are defined in terms of volume-averaged fluxes and surface-averaged net or partial currents.
2. The volume (node) averaged fluxes and surface-averaged currents are related through auxiliary relationships. Such relationships, in the case of modern nodal methods, are often obtained via a transverse integration procedure.
3. Transverse leakage terms appear due to the transverse integration procedure and these are approximated in some way. Typical approaches would include the "flat leakage" approximation and the "quadratic leakage" approximation. The latter introduces a three node quadratic fit for the transverse leakage term in the transversely-integrated equations and has become the industry standard in Cartesian geometry.

Beyond these similarities, methods differ largely in the form of the intra-node solution. Two classes of methods which are most often utilized are the Analytic Nodal Method (ANM) and the Nodal Expansion Method (NEM). In the case of the analytic method, the intra-node flux shape is the analytic solution of the one-dimensional transversely-integrated diffusion equation. This approach requires no approximations other than the transverse leakage approximation mentioned in point three above. In one dimension, therefore, the analytic method is formally exact. In the nodal expansion method the solution of the auxiliary one-dimensional equations is approximated by a polynomial expansion (typically fourth-order).

In both the ANM and NEM approaches, which constitute the most widely used nodal approaches, the quadratic transverse leakage approximation (QLA) is the major source of inaccuracy, but also of convergence problems. This approach uses information from the direction under consideration to construct the leakage shape from the transverse directions. This implies a certain level of long-range coupling between nodes and a statement regarding the quadratic transverse leakage approximation is often made that it works well in checkerboard-type material arrangements. Typically, the simple three-node fit breaks down in the following cases:

- Near the core/reflector interface of the reactor and typically in reflector nodes near the boundary where the flux gradient due to thermalization is sharper than that which the quadratic approximation can match. In such cases the numerical scheme can lead to negative fluxes, which in turn may destabilize the entire calculation and lead to non-convergence (Smith, 1979). In these reflector areas, node-averaged errors in excess of 10% in the nodal flux are not unusual for the quadratic leakage approximation and may be substantially larger if convergence problems occur. There is no natural extension of the three-node fit to higher leakage orders;
- At boundary nodes, where a three node fit will suffer, since no average leakage may be consistently defined for boundary nodes with no neighbors. In this case some approximate leakage, dependent on the boundary conditions of the system, is typically utilized in a fictitious outer node. Alternatively, a two node linear fit could be utilized in order to avoid the boundary problem;
- At interfaces with sharp material changes, as would be the case near control rod positions (Ougouag and Rajić, 1988). In turn, such errors may lead to the misprediction of important safety parameters such as control rod worth. At such interfaces, errors in node-averaged power distribution in excess of 2% could be found for difficult problems. Errors in node-averaged flux distribution may be substantially higher; and
- In adjacent nodes which have very different sizes, or within nodes with large aspect ratios. This deficiency places a restriction on the code user with respect to the choice of meshing scheme and in extreme cases, leads to non-convergence of the solution.

These situations occur regularly in power reactors and especially in research reactor cores with compact, heterogeneous designs.

1.2. Higher-order nodal methods

Higher-order nodal methods were first developed from the perspective of creating consistent nodal diffusion methods. In this sense the property of consistency is defined such that all numerical approximations result from the basic discretization method. Thus, the numerical solution approaches the reference solution when either the mesh size is decreased or the approximation order is increased. The application of the quadratic leakage approximation clearly violates this principle and some of the earliest efforts to address this were by Dorning (1979) and Dilbert and Lewis (1985). The first significant step in formulating a consistent, transversely-integrated, higher-order nodal method was achieved by Ougouag and Rajić (1988). In the work by Ougouag and Rajić (1988), a coupled set of auxiliary one-dimensional higher-order nodal diffusion equations were generated via the process of weighted transverse integration. This formalism, which is akin to the class of weighted residual methods (Nakamura, 1977),