A new power mapping method based on ordinary kriging and determination of optimal detector location strategy

Xingjie Peng [a,b,*], Kan Wang [a], Qing Li [b]

[a] Department of Engineering Physics, Tsinghua University, Beijing, China
[b] Science and Technology on Reactor System Design Technology Laboratory, Nuclear Power Institute of China, Chengdu, China

ABSTRACT

The Ordinary Kriging (OK) method is presented that is designed for a core power mapping calculation of pressurized water reactors (PWRs). Measurements from DayaBay Unit 1 PWR are used to verify the accuracy of the OK method. The root mean square (RMS) reconstruction errors are kept at less than 0.35%, and the maximum reconstruction relative errors (RE) are kept at less than 1.02% for the entire operating cycle. The reconstructed assembly power distribution results show that the OK method is fit for core power distribution monitoring. The quality of power distribution obtained by the OK method is partly determined by the neutron detector locations, and the OK method is also applied to solve the optimal neutron detector location problem. The spatially averaged ordinary kriging variance (AOKV) is minimized using simulated annealing, and then, the optimal in-core neutron detector locations are obtained. The result shows that the current neutron detector location of DayaBay Unit 1 reactor is near-optimal.

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1. Introduction

Core power distribution monitoring in operating power reactors is very important in core surveillance, the 3-D power distribution is one of the basic operation parameters which can determine many other important parameters such as power peaking factor, enthalpy rising factor and quadrant tilt ratio used to evaluate the operation condition of reactor and the safe margin. The economy of reactor could be optimized if the real time 3-D power distribution is well obtained and used for surveillance and regulation. Most commercial power reactors in operation are equipped with fixed or movable in-core neutron detectors to obtain power distribution information. Many kinds of on-line monitoring systems, such as BEACON (Boyd and Miller, 1996) and GNF-ARGOS (Tojo et al., 2008), have been developed to estimate in-core power distributions using fixed in-core detectors. The detector results at certain locations reflect the actual reactor flux or power can be applied to improve the results of the only diffusion calculations.

Several computational methods have been proposed for power or flux mapping. The CANDU on-line flux mapping system (Tang et al., 1978) converts the 102 vanadium detector signals to thermal fluxes at the detector sites and then maps out the 3D flux distribution by a process of least-squares fitting of the measured

thermal fluxes to a linear expansion of pre-calculated flux modes. Combustion Engineering (CE) nuclear power plants use the Combustion Engineering Core (CECOR) Karlson, 1995 method to estimate the power distributions, and the pre-calculated two-dimensional coupling coefficients are used. Jang et al. (2004) proposed a three-dimensional coupling coefficients method and Webb and Brittingham (2000) proposed a Lagrange multiplier method, which both can be regarded as an improved version of the CECOR method. Lee and Kim (2003) proposed a least-squares method by combining the coarse mesh finite difference (CMFD) form of the fixed-source diffusion equation and the detector response equation to form an over-determined linear equation. Zhong et al. (2010) proposed a mixed method based on the least-squares method and the harmonics synthesis method. Li et al. (2013) proposed three methods, namely, weight coefficient method, polynomial expand method and thin plane spline method to fit the deviation between measured and predicted results for two-dimensional radial plane.

The quality of maps obtained by interpolation of observations of a target spatial variable at a restricted number of locations, is partly determined by the spatial pattern of the detector locations. In order to get the best possible results from the power mapping reconstruction procedure, the detector locations need to be optimized. The algorithm for optimal placement of detectors appear to be very similar regardless of the application. All can be posed as selecting a subset of locations from a large set of candidate locations. A natural approach to solve this problem is to minimize the prediction error variance by using effective combinatorial

optimization methods like genetic algorithm and simulated annealing algorithm.

The ordinary kriging method of interpolation in geostatistical analysis is applied to many different subjects (Pokhrel et al., 2013), such as precipitation, elevation, air temperature, soil properties and electrical conductivities. In nuclear field, Lockwood and Anitescu (2012) investigate the issue of providing a statistical model for the response of a computer model of nuclear engineering system for use in uncertainty propagation. In this paper, we introduce the ordinary kriging method into nuclear reactor power distribution monitoring. A new power mapping method based on ordinary kriging method is presented, and then, the spatially averaged ordinary kriging variance is minimized using the well known simulated annealing algorithm (Brus et al., 2013) based on a Metropolis–Hastings one to obtain the optimal detector locations.

2. Methodology

2.1. Ordinary kriging method

Kriging is a group of geostatistical techniques (Pokhrel et al., 2013) to interpolate the value of a random field at an unobserved location from observations of its value at nearby locations. It belongs to a family of linear least squares estimation algorithms that are used in several geostatistical applications. Linear kriging is divided into simple kriging (known mean), ordinary kriging (unknown but constant mean) and universal kriging (the mean is an unknown linear combination of known functions), depending on the mean value specification. We shall restrict the discussion here to ordinary kriging. Ordinary kriging is widely used because it is statistically the best linear unbiased estimator. Ordinary kriging is linear because its estimates are linear combination of the available data. It is unbiased because it attempts to keep the mean residual to be zero. Finally, it is called best because it tries to minimize the residual variance.

Let the data be sampled at N locations $(x_1, \ldots, x_N)$, and the corresponding values be $(v_1, \ldots, v_N)$. The value $\hat{v}_j$ at an unknown location $\hat{x}_j$ is estimated as a weighted linear combination of sampled values, given by,

$$\tilde{v}_j = \sum_{i=1}^{N} w_i v_i \tag{1}$$

Here, $\tilde{v}_j$ is the estimate, and let $\hat{v}_j$ be the actual value (unknown) at $\hat{x}_j$. To find the weights, the values (v_i and $\hat{v}_j$) are assumed to be stationary random functions,

$$E[v_i] = E[\hat{v}_j] = E(v) \tag{2}$$

For unbiased estimates, we can get

$$
\begin{aligned}
& E[v_i] = E[\hat{v}_j] = E(v) \\
& E[\hat{v}_j - \tilde{v}_j] = 0 \\
& E[\hat{v}_j] - E[\tilde{v}_j] = 0 \\
& E[\hat{v}_j] - E\left[\sum_{i=1}^{N} w_i v_i\right] = 0 \\
& E[\hat{v}_j] - \sum_{i=1}^{N} w_i E[v] = 0 \\
& \sum_{i=1}^{N} w_i = 1
\end{aligned}
\tag{3}
$$

Let the residue be r_j

$$r_j = \tilde{v}_j - \hat{v}_j \tag{4}$$

Therefore, the residual variance is given by,

$$Var(r_j) = Cov\{\tilde{v}_j \tilde{v}_j\} - 2Cov\{\tilde{v}_j \hat{v}_j\} + Cov\{\hat{v}_j \hat{v}_j\} \tag{5}$$

The first term can further be simplified as follow,

$$
\begin{aligned}
Cov\{\tilde{v}_j \tilde{v}_j\} &= Var\{\tilde{v}_j\} = Var\left\{\sum_{i=1}^{N} w_i v_i\right\} \\
&= \sum_{i=1}^{N}\sum_{k=1}^{N} w_i w_k Cov\{v_i v_k\} = \sum_{i=1}^{N}\sum_{k=1}^{N} w_i w_k \widehat{C}_{ik}
\end{aligned}
\tag{6}
$$

The second term can be written as,

$$
\begin{aligned}
Cov\{\tilde{v}_j \hat{v}_j\} &= Cov\left\{\left(\sum_{i=1}^{N} w_i v_i\right)\hat{v}_j\right\} = \sum_{i=1}^{N} w_i Cov\{v_i \hat{v}_j\} \\
&= \sum_{i=1}^{N} w_i \widehat{C}_{i0}
\end{aligned}
\tag{7}
$$

Finally, the third term can be expressed as

$$Cov\{\hat{v}_j \hat{v}_j\} = \widehat{C}_{00} \tag{8}$$

Substituting from Eqs. (6)–(8) in Eq. (5),

$$Var(r_j) = \sum_{i=1}^{N}\sum_{k=1}^{N} w_i w_k \widehat{C}_{ik} - 2\sum_{i=1}^{N} w_i \widehat{C}_{i0} + \widehat{C}_{00} \tag{9}$$

For ordinary kriging, it is required to find w by minimizing $Var(r_j)$ with respect to w subject to the constraint Eq. (3). This can be written as the minimization of the penalized cost function,

$$J(w) = \sum_{i=1}^{N}\sum_{k=1}^{N} w_i w_k \widehat{C}_{ik} - 2\sum_{i=1}^{N} w_i \widehat{C}_{i0} + \widehat{C}_{00} + 2\lambda\left(\sum_{i=1}^{N} w_i - 1\right) \tag{10}$$

with 2λ the Lagrange multiplier. Taking derivatives of J with respect to w and λ,

$$\frac{\partial J}{\partial w_i} = 2\sum_{k=1}^{N} w_k \widehat{C}_{ik} - 2\widehat{C}_{i0} + 2\lambda \tag{11}$$

$$\frac{\partial J}{\partial \lambda} = 2\left(\sum_{i=1}^{N} w_i - 1\right) \tag{12}$$

Setting this to zero, we get the following system to solve, to obtain the weights w and λ,

$$
\begin{pmatrix}
\widehat{C}_{11} & \cdots & \widehat{C}_{1N} & 1 \\
\vdots & \ddots & \vdots & \vdots \\
\widehat{C}_{N1} & \cdots & \widehat{C}_{NN} & 1 \\
1 & \cdots & 1 & 0
\end{pmatrix}
\begin{pmatrix}
w_1 \\
\vdots \\
w_N \\
\lambda
\end{pmatrix}
=
\begin{pmatrix}
\widehat{C}_{10} \\
\vdots \\
\widehat{C}_{N0} \\
1
\end{pmatrix}
\tag{13}
$$

Substituting Eq. (13) into Eq. (9), we can derive the kriging variance,

$$
\begin{aligned}
Var(r_j) &= \sum_{i=1}^{N}\sum_{k=1}^{N} w_i w_k \widehat{C}_{ik} - 2\sum_{i=1}^{N} w_i \widehat{C}_{i0} + \widehat{C}_{00} \\
&= \sum_{i=1}^{N} w_i\left(\sum_{k=1}^{N} w_k \widehat{C}_{ik} - 2\widehat{C}_{i0}\right) + \widehat{C}_{00} \\
&= \sum_{i=1}^{N} w_i\left(\widehat{C}_{i0} - \lambda - 2\widehat{C}_{i0}\right) + \widehat{C}_{00} \\
&= \widehat{C}_{00} - \sum_{i=1}^{N} w_i \widehat{C}_{i0} - \sum_{i=1}^{N} w_i \lambda = \widehat{C}_{00} - \sum_{i=1}^{N} w_i \widehat{C}_{i0} - \lambda
\end{aligned}
\tag{14}
$$

Eq. (13) needs to be solved at each unknown location $\hat{x}_j$.

2.2. Covariance functions

There are two ways of specifying the covariance $\widehat{C}_{ij}$, either by a standard function or by evaluating it empirically at each location. A