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Hybrid subgroup decomposition method for solving fine-group eigenvalue transport problems

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ABSTRACT

In this paper, a new hybrid method for solving fine-group eigenvalue transport problems is developed. This method extends the subgroup decomposition method to efficiently couple a new coarse-group quasi transport theory with a set of fixed-source transport decomposition sweeps to obtain the fine-group transport solution. The advantages of the quasi transport theory are its high accuracy, straight-forward implementation and numerical stability. The hybrid method is analyzed for a 1D benchmark problem characteristic of boiling water reactors (BWR). It is shown that the method reproduces the fine-group transport solution with high accuracy while increasing the computational efficiency up to 12 times compared to direct fine-group transport calculations.

1. Introduction

Recently, Douglass and Rahnema (Douglass and Rahnema, 2012a) developed a new subgroup decomposition method (SGD) for treating the energy variable in the Boltzmann equation. This work improved the consistent generalized energy condensation theory (Douglass and Rahnema, 2012b) by extending the cross section condensation process to preserve spectral accuracy in condensed-group transport calculations in a simpler and more direct manner. That is, cross sections and the angular flux are no longer expanded in energy. The SGD method directly couples a consistent coarse-group criticality calculation with a set of fixed-source decomposition sweeps to obtain the fine-group spectrum without the need to solve for energy moments of the flux. The SGD method was recently extended to diffusion theory (Yasseri and Rahnema, 2013a). The transport theory SGD (TSGD) and diffusion theory SGD (DSGD) methods utilize coarse-group transport/diffusion calculations and a set of transport/diffusion decomposition sweeps to unfold the fine-group flux spectrum (solution). The key feature of both SGD methods is the ability to correct for spectral (core environment) effects resulting from energy condensation. Consistent multi-group formulations (*i.e.*, energy-angle coupling) and on-the-fly cross section recondensation at the core level are the main reasons that the SGD methods reproduce the fine-group solution independent of the coarse-group structure.

Anistratov and Gol'din (2011) developed a multi-level method in 1D slab geometries for solving multi-group eigenvalue transport problems. In their work, a combination of effective one-group low-order quasi-diffusion equation and multi-group low-order equation are utilized to accelerate the multi-group k-eigenvalue transport problems. In the multi-level approach, the multi-group low order quasi-diffusion is used to accelerate the multi-group transport in the first layer and an effective gray (one-group) low order quasi-diffusion is utilized to accelerate the multi-group low order equation in the second layer by evaluating the eigenvalue and one-group flux. In this work, the accuracy of the eigenvalue for the multi-level approach compared to multi-group transport solution for different spatial and angular discretization is demonstrated. However, no comparison of the flux spectrum and its accuracy using the multi-level approach is shown. In a different work by Anistratov (2011), a nonlinear diffusion acceleration (NDA) method is presented as a fast iterative algorithm for solving multi-group eigenvalue transport problems. This work demonstrates that a one-group low order NDA consistent with eigenvalue transport problems accelerates the multi-group low order NDA equations. It is shown that the eigenvalue and flux converge after a few iterations. The paper does not discuss the speed up factor nor the accuracy of the method. That is, it is not shown whether the method reproduces the fine-group transport solution or not.

The TSGD method can also be viewed as an acceleration method for solving heterogeneous fine-group eigenvalue transport problems by using coarse-group transport calculation iteratively. This will be demonstrated in this paper. Since decomposition sweep (a step in the SGD method that globally unfolds the flux

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spectrum from the coarse-group flux) depends on the coarse-group flux and eigenvalue, coarse-group diffusion is an efficient candidate for replacing the coarse-group transport calculation. However, the standard coarse-group diffusion theory is not capable of achieving transport accuracy especially in regions where flux behavior is highly anisotropic such as high absorbing regions, near boundaries and localized sources. Hence, a theory that is computationally as efficient as diffusion theory but as accurate as transport theory is highly desirable. To this end, a new coarse-group "quasi transport" theory is developed in this paper. The advantage of the new method is that angular details are embedded in an additional cross section while retaining the standard form of the diffusion coefficient. As a result, the quasi transport method can be easily implemented into existing fine-mesh diffusion codes by introducing only one additional term. In this paper, the SGD method is combined with the coarse-group quasi transport theory resulting in a hybrid transport/quasi transport SGD (HSGD) method to further increase computational efficiency while maintaining fine-group transport accuracy.

In Section 2.1, the new coarse-group quasi transport theory is derived in general geometry and in Section 2.2 the HSGD method is described with the quasi transport theory. The accuracy of the hybrid method is investigated in a 1D BWR benchmark problem in Section 3. Concluding remarks and future work are found in Section 4. Spatial discretization schemes for the quasi transport theory in 1D slab geometry are included in Appendix A.

2. Method

2.1. Quasi transport theory

For an eigenvalue problem, the fine-group transport angular flux is governed by Eq. (1) in which G is the total number of fine groups $\{g \mid g = 1, 2, 3, \ldots, G\}$.

$$\begin{aligned}\hat{\Omega}\cdot\vec{\nabla}\psi^g(\vec{r},\hat{\Omega})+\sigma^g(\vec{r})\psi^g(\vec{r},\hat{\Omega}) &= \sum_{g'=1}^{G}\int_{4\pi}d\hat{\Omega}'\frac{1}{4\pi}\left\{\sigma_{s0}^{g'\to g}(\vec{r})\right.\\ &\left.+3\sigma_{s1}^{g'\to g}(\vec{r})\hat{\Omega}\cdot\hat{\Omega}'\right\}\psi^{g'}(\vec{r},\hat{\Omega}')\\ &+\frac{\chi^g}{4\pi k}\sum_{g'=1}^{G}\nu\sigma_f^{g'}(\vec{r})\phi^{g'}(\vec{r})\end{aligned}\tag{1}$$

Standard notation (Douglass and Rahnema, 2012a) is used in Eq. (1). The fission and scattering kernels are assumed isotropic and linearly anisotropic, respectively. These assumptions are common in lattice depletion and core analysis methods for thermal (water) reactor systems.

Let C be the number of coarse groups where any fine-group h is fully contained in coarse-group c. By integrating/summing over the energy range contained in coarse-group c, the coarse-group transport equation is obtained as shown in Eq. (2). The more general form including arbitrary scattering kernel is found in Douglass and Rahnema (2012a).

$$\begin{aligned}&\hat{\Omega}\cdot\vec{\nabla}\psi^c(\vec{r},\hat{\Omega})+\sigma^c(\vec{r})\psi^c(\vec{r},\hat{\Omega})\\ &=\sum_{c'=1}^{C}\left\{\frac{\sigma_{s0}^{c'\to c}(\vec{r})}{4\pi}+\hat{\Omega}\cdot\frac{3}{4\pi}\sigma_{s1}^{c'\to c}(\vec{r})\right\}\int_{4\pi}d\hat{\Omega}'\psi^{c'}(\vec{r},\hat{\Omega}')\\ &+\frac{\chi^c}{4\pi k^c}\sum_{c'=1}^{C}\nu\sigma_f^{c'}(\vec{r})\phi^{c'}(\vec{r})+\delta_1^c(\vec{r},\hat{\Omega})\phi^c(\vec{r})\end{aligned}\tag{2}$$

where the coarse-group coefficients are defined in the following equations:

$$\psi^c(\vec{r},\hat{\Omega})=\sum_{h\in c}\psi^h(\vec{r},\hat{\Omega})\tag{3}$$

$$\sigma^c(\vec{r})=\frac{\sum_{h\in c}\sigma^h(\vec{r})\phi^h(\vec{r})}{\sum_{h\in c}\phi^h(\vec{r})}\tag{4}$$

$$\sigma_{s0}^{c'\to c}(\vec{r})=\frac{\sum_{h\in c}\sum_{h'\in c'}\sigma_{s0}^{h'\to h}\phi^{h'}(\vec{r})}{\sum_{h'\in c'}\phi^{h'}(\vec{r})}\tag{5}$$

$$\vec{\sigma}_{s1}^{c'\to c}(\vec{r})=\frac{\sum_{h\in c}\sum_{h'\in c'}\sigma_{s1}^{h'\to h}(\vec{r})\vec{J}^{h'}(\vec{r})}{\sum_{h'\in c'}\phi^{h'}(\vec{r})}\tag{6}$$

$$\chi^c=\sum_{h\in c}\chi^h\tag{7}$$

$$\nu\sigma_f^{c'}(\vec{r})=\frac{\sum_{h'\in c'}\nu\sigma_f^{h'}(\vec{r})\phi^{h'}(\vec{r})}{\sum_{h'\in c'}\phi^{h'}(\vec{r})}\tag{8}$$

$$\delta_1^c(\vec{r},\hat{\Omega})=\frac{\sum_{h\in c}[\sigma^c(\vec{r})-\sigma^h(\vec{r})]\psi^h(\vec{r},\hat{\Omega})}{\sum_{h\in c}\phi^h(\vec{r})}\tag{9}$$

Taking the 0th and 1st angular moment of Eq. (2) will result in Eqs. (10) and (11).

$$\vec{\nabla}\cdot\vec{J}^c(\vec{r})+\sigma^c(\vec{r})\phi^c(\vec{r})=\sum_{c'=1}^{C}\sigma_{s0}^{c'\to c}(\vec{r})\phi^{c'}(\vec{r})+\frac{\chi^c}{k^c}\sum_{c'=1}^{C}\nu\sigma_f^{c'}(\vec{r})\phi^{c'}(\vec{r})\tag{10}$$

$$\vec{\nabla}\cdot\overleftrightarrow{\Pi}^c(\vec{r})+\sigma_{tr}^c(\vec{r})\vec{J}^c(\vec{r})=\left\{\vartheta_2^c(\vec{r})+\int_{4\pi}d\hat{\Omega}\hat{\Omega}\vartheta_1^c(\vec{r},\hat{\Omega})\right\}\phi^c(\vec{r})\tag{11}$$

where

$$\overleftrightarrow{\Pi}^c(\vec{r})=\int_{4\pi}d\hat{\Omega}\hat{\Omega}\hat{\Omega}\psi^c(\vec{r},\hat{\Omega})\tag{12}$$

$$\vartheta_1^c(\vec{r},\hat{\Omega})=\frac{\sum_{h\in c}[\sigma_{tr}^c(\vec{r})-\sigma_{tr}^h(\vec{r})]\psi^h(\vec{r},\hat{\Omega})}{\sum_{h\in c}\phi^h(\vec{r})}\tag{13}$$

and

$$\vartheta_2^c(\vec{r})=\frac{\sum_{c'=1}^{C}\sum_{h'\in c'}\sum_{h\in c}[\sigma_{s1}^{h'\to h}(\vec{r})\vec{J}^{h'}(\vec{r})-\sigma_{s1}^{h\to h'}(\vec{r})\vec{J}^{h}(\vec{r})]}{\sum_{h\in c}\phi^h(\vec{r})}\tag{14}$$

In the above equations, the vector symbol for current, gradient and current weighted cross sections has been explicitly expressed. The double sided arrow (↔) indicates a tensor. For the ease of implementation, Eq. (11) is modified as below to resemble the standard coarse-group diffusion equation while maintaining higher order transport effects (beyond 1st order in angle).

$$\begin{aligned}\frac{1}{3}\vec{\nabla}\phi^c(\vec{r})+\sigma_{tr}^c(\vec{r})\vec{J}^c(\vec{r}) &= \left\{\vec{\vartheta}_2^c(\vec{r})+\int_{4\pi}d\hat{\Omega}\hat{\Omega}\vartheta_1^c(\vec{r},\hat{\Omega})\right\}\phi^c(\vec{r})\\ &+\vec{\vartheta}_3^c(\vec{r})\phi^c(\vec{r})\end{aligned}\tag{15}$$

The new term in the above equation is defined as:

$$\vec{\vartheta}_3^c(\vec{r})=\frac{\sum_{h\in c}\left[\frac{1}{3}\vec{\nabla}\phi^h(\vec{r})-\vec{\nabla}\cdot\overleftrightarrow{\Pi}^h(\vec{r})\right]}{\sum_{h\in c}\phi^h(\vec{r})}\tag{16}$$

Combining Eqs. (10)–(16) yield the following coarse-group quasi transport equations.

$$\begin{aligned}\vec{\nabla}\cdot\vec{J}^c(\vec{r})+\sigma^c(\vec{r})\phi^c(\vec{r}) &= \sum_{c'=1}^{C}\sigma_{s0}^{c'\to c}(\vec{r})\phi^{c'}(\vec{r})\\ &+\frac{\chi^c}{k^c}\sum_{c'=1}^{C}\nu\sigma_f^{c'}(\vec{r})\phi^{c'}(\vec{r})\end{aligned}\tag{17a}$$