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Parameter analysis of the neutron point kinetics equations with feedback temperature effects

Mohamed S. El_Tokhy *, Imbaby I. Mahmoud

Engineering Department, NRC, Atomic Energy Authority, P.No. 13759, Inshas, Egypt

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ABSTRACT

The paper presents a solution technique for modeling point nuclear kinetic equations based on one group of delayed neutrons and temperature feedback. Explicit and implicit solutions for point kinetic equations are used for this purpose. Analytical treatment is conducted. Mathematical models describing neutron density and reactor reactivity are deduced. Also, Matlab Simulink and VisSim environments are used to achieve the implicit solution. Using of graphical user interface allows a quick experimentation with alternative values of performance parameters such as initial reactivity, initial neutron flux density, temperature coefficient of reactivity and reciprocal of thermal capacity of reactor. Moreover, implicit solutions of dynamic equations governing point nuclear kinetic equations provide exact handling of the device performance. Proposed mathematical models and block diagram simulation results are validated against published work and full agreements are obtained. Several performance parameters are tuned to enhance the performance of these point nuclear kinetic equations through the presented methodology. The resultant performance characteristics and comparison among investigated models are presented in this work. The obtained results confirm that the implicit solution showed less accurate representation of the studied point nuclear kinetic equations compared to mathematical models. Furthermore, the effect of control rod on the neutron flux density was discussed.

1. Introduction

The model of the point reactor kinetics equations is one of the most important reduced models of nuclear science and engineering (Yamoah et al., 2013). It has been the subject of countless studies and applications to understand the neutron dynamics and its effects (Yamoah et al., 2013). The point reactor kinetics equations are a system of coupled non-linear ordinary differential equations (Aboanber and Hamada, 2003; Tashakor et al., 2010) which describe the neutron density and the delayed neutron precursor concentrations in the system of nuclear reactor (Ray and Patra, 2013). These equations can be obtained from basic principles and can be precisely derived from the transport or diffusion equation as in (Aboanber, 2003a). All these derivations in (Aboanber, 2003a) stated that the equations with one equivalent group are analytically solved for certain simple forms of reactivity changes. In an accurate reactor kinetics calculation, it is necessary to consider in detail the production and decay of each of the six groups of delayed neutron precursors (Aboanber, 2003a).

The density of neutron flux and the delayed neutron precursor concentration are the important parameters to be studied safety and the transient behavior of the reactor power. They determine the time-dependent behavior of the power level of a nuclear reactor (Hamieh and Saidinezhad, 2012). Also, they are influenced by control rod position (Hamieh and Saidinezhad, 2012). Although, it is important to study the reactor kinetics equations as a function of space and time, some tightly coupled reactor can be analyzed with point kinetics equations (Nahla, 2010) .

The reactivity is one of the most important properties in a nuclear reactor due to that it is directly related to the control of the reactor (Polo-Labarrios and Espinosa-Paredes, 2012). The reactivity is basically inserted when the control rods are lifted. In practice each step of the lifting of the control rods is a step introducing linear reactivity in a certain period of time. However, the control rods are lifted discontinuously, and when near to criticality the length of each step is a time interval which allows the duration of a transitory so that the reactor can reach criticality in a slow and safety way (Polo-Labarrios and Espinosa-Paredes, 2012). However, uncontrolled control rods withdrawal or ejection is the most common type of initiator for a reactivity insertion accident (Polo-Labarrios and Espinosa-Paredes, 2012).

Several methods have been described in literature for investigating and solution of point-reactor neutron kinetics equations stated in (Aboanber, 2009; Chen et al., 2007; Ray and Patra, 2013; Tashakor et al., 2010; Wahi and Kumawat, 2011). Lately, an increasing interest in the task of searching for analytical solutions of linear and non-linear problems by the scientific community has

* Corresponding author. Tel.: +20 1068395079.
E-mail address: engtokhy@gmail.com (M.S. El_Tokhy).

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been noticed (Petersen et al., 2011). Analytical solutions of the point-reactor kinetics equations provide insight into the dynamics of nuclear reactor operation and are useful, for example, in understanding the power fluctuations experienced during start-up or shut-down when the control rods are adjusted (Hamieh and Saidinezhad, 2012). Furthermore, the analytical and numerical solution of the point kinetics equations in the presence of temperature feedback is useful for estimating the transient behavior of the reactor power and the parameters of the reactor core (Tashakor et al., 2010). The advantage of compact analytical solutions is that they give better insight into the properties of the system (Pazsit et al., 2012). Also, they can be used to predict the behavior in limiting cases such as the reactor kinetic approximations (Pazsit et al., 2012). Further, the analytical solution in some sense, eliminates or at least mitigates the difficulty of the mathematical task of the error evaluation required by numerical methods, except for the round-off error (Petersen et al., 2011). This paper is organized in the following manner. First, the basic assumptions and model description of this approach is described in Section 2. Then, in Section 3 the performance analysis of the present investigation is presented. Discussions of the results are expressed in Section 4. Finally, concluding remarks are summarized in Section 5.

2. Basic assumptions and model description

The one-group diffusion model is adopted, focusing the attention specifically on the spatial aspects. Also, the treatment of the energy variable through a multi-group approximation is a very interesting task of nuclear reactor kinetics. However, due to the additional complication introduced into the mathematical formalism (Corno et al., 2008), this aspect cannot be included in the present work. For the reactor operating at steady output power, the effect of the extraneous neutron resource can be neglected. Assume that a generic reactor has negative temperature coefficient of reactivity where the initial reactivity is less than the total fraction of delayed neutrons. It was recognized that the nuclear reactor was mathematically a very complex device, particularly if one were to treat the kinetic effects on a spatial basis throughout the reactor. Soodak and Campbell (Aboanber and El Mhlawy, 2009) first indicated what the response of the reactor would be if all of the spatial effects were lumped and the reactor were considered as a simple entity, all portions behaving the same way at the same time (Aboanber and El Mhlawy, 2009). That was recently confirmed by many authors (Aboanber and El Mhlawy, 2009). The equations for the response of the neutron level in a reflected reactor to any change in reactivity were described in the space-average approximation by a coupled system of stiff differential equations for two-point kinetics equations (the core region and the reflector region) (Aboanber and El Mhlawy, 2009). The heat loss is assumed to be negligible (such as in a very rapid transient) to consider for fuel temperature coefficient of reactivity. In other words, under simplified conditions, an analytical solution to the point reactor kinetics equations can be obtained, for example, such as there being no extraneous neutron source, assuming a prompt jump approximation (Yamoah et al., 2013) and a constant neutron source.

3. Mathematical and block diagram models

A detailed analysis for the proposed model is considered below. The point-reactor neutron kinetics equations with one group of delayed neutrons and without the effect of extraneous neutron source are given as follows (Aboanber, 2003a, 2003b; Aboanber and Hamada, 2003; Chen et al., 2007; Espinosa-Paredes et al., 2011; Hamieh and Saidinezhad, 2012; Li et al., 2009; Nahla, 2010; Nahla and Zayed, 2010; Petersen et al., 2011; Ray and Patra, 2013; Tashakor et al., 2010; Vyawahare and Nataraj, 2013; Wahi and Kumawat, 2011; Yamoah et al., 2013)

$$\frac{dn_t}{dt} = \frac{\rho_t - \beta}{l} n_t + \lambda C_t \tag{1}$$

$$\frac{dC_t}{dt} = \frac{\beta}{l} n_t - \lambda C_t \tag{2}$$

where n_t, t, ρ_t, β, l, λ, and C_t are represented as the neutron density representing the reactor power level, the time, the reactivity, the total fraction of the six group of delayed neutrons, the prompt neutron lifetime, the radioactive decay constant of delayed neutron precursors, and the average density of delayed neutron precursors, respectively. Eq. (1) can be arranged and rewritten as follows:

$$\frac{dn_t}{dt} = \frac{\rho_t}{l} n_t - \frac{\beta}{l} n_t + \lambda C_t \tag{3}$$

By differentiation of Eq. (3) with respect to time, we get

$$\frac{d^2 n_t}{dt^2} = \frac{\rho_t}{l}\frac{dn_t}{dt} + \frac{n_t}{l}\frac{d\rho_t}{dt} - \frac{\beta}{l}\frac{dn_t}{dt} + \lambda\frac{dC_t}{dt} \tag{4}$$

But, from Eq. (4), the term $\lambda\frac{dC_t}{dt}$ can be rewritten as follows:

$$\lambda\frac{dC_t}{dt} = \lambda\left(\frac{\beta}{l} n_t - \lambda C_t\right) \tag{5}$$

Substituting from Eq. (5) in Eq. (4) yields

$$\frac{d^2 n_t}{dt^2} = \frac{\rho_t}{l}\frac{dn_t}{dt} + \frac{n_t}{l}\frac{d\rho_t}{dt} - \frac{\beta}{l}\frac{dn_t}{dt} + \lambda\frac{\beta}{l} n_t - \lambda^2 C_t \tag{6}$$

One of the important properties in a nuclear reactor is the reactivity, due to the fact that it is directly related to the control of the reactor (Yamoah et al., 2013). The start-up process of a nuclear reactor requires that reactivity is varied in the system by lifting the control rods discontinuously. In practice, the control rods are withdrawn at time intervals such that reactivity is introduced in the reactor core linearly, to allow criticality to be reached in a slow and safe manner (Yamoah et al., 2013). Change of reactivity with time is given by (Aboanber, 2009; Chen et al., 2007; Hamieh and Saidinezhad, 2012; Nahla and Zayed, 2010):

$$\frac{d\rho_t}{dt} = -\alpha K_c n_t \tag{7}$$

where K_c and α denote the reciprocal of thermal capacity of reactor and a negative temperature coefficient of reactivity, respectively. By substituting from both Eq. (1) and Eq. (7) in Eq. (6), we find

$$\frac{d^2 n_t}{dt^2} = \frac{\rho_t}{l}\left(\frac{\rho_t - \beta}{t} n_t + \lambda C_t\right) + \frac{n_t}{l}(-\alpha K_c n_t) - \frac{\beta}{l}\left(\frac{\rho_t - \beta}{t} n_t + \lambda C_t\right) + \lambda\frac{\beta}{l} n_t - \lambda^2 C_t \tag{8}$$

However, $\lambda^2 C_t$ is very small quantity, so it can be neglected (Hamieh and Saidinezhad, 2012) and Eq. (8) was rewritten as follows:

$$\frac{d^2 n_t}{dt^2} = \frac{1}{l}(\rho_t - \beta)\frac{dn_t}{dt} - \frac{n_t}{l}(\lambda\beta + \alpha K_c n_t) \tag{9}$$

With the neutron jump approximation (Chen et al., 2007), Eq. (9) can be simplified and arranged as:

$$\frac{dn_t}{dt} = \frac{n_t}{\rho_t - \beta}(\lambda\beta + \alpha K_c n_t) \tag{10}$$

Since, $\frac{dn_t}{dt} = \frac{dn_t}{d\rho_t}\frac{d\rho_t}{dt}$, thus, Eq. (10) can be arranged as follows:

$$\frac{dn_t}{d\rho_t}\frac{d\rho_t}{dt} = \frac{n_t}{\rho_t - \beta}(\lambda\beta + \alpha K_c n_t) \tag{11}$$