Annals of Nuclear Energy 67 (2014) 109–128

Contents lists available at ScienceDirect

Annals of Nuclear Energy

journal homepage: www.elsevier.com/locate/anucene

Black-box system identification for reduced order model construction

Wolfgang Polifke *

Fachgebiet für Thermodynamik, Technische Universität München, Boltzmannstr. 15, D-85747 Garching, Germany

ARTICLE INFO

Article history:
Received 20 May 2013
Accepted 22 October 2013
Available online 27 December 2013

Keywords:
System identification
Model reduction
Fluid dynamics
Aero- and thermoacoustics
Multi-physics
Multi-scale

ABSTRACT

For the simulation of multi-physics, multi-scale phenomena, it is often advantageous to build a comprehensive system- or process-model from a collection of sub-models, each of them purposely constructed to describe a certain aspect of the overall problem with high accuracy at low computational cost. Such strategies of *divide et impera* ("divide and conquer") integrate modeling approaches of different complexity for different phenomena and scales. *Reduced order models* (ROMs) identified from time series data can play an important part in such a scheme.

The present paper reviews a body of work in aero- and thermo-acoustics, where computational fluid dynamics (CFD) simulation is combined with tools from system identification to characterize the dynamic response of a sub-system (an "element") to incoming flow perturbations. The element under consideration is treated as a "black box" with a given structure of inputs and outputs. In general, multiple inputs and multiple outputs are present (MIMO model), in the simplest case only a single input and a single output need be considered (SISO structure). Once the response to a broad-band excitation signal is determined by numerical simulation, a ROM representation of the element dynamics can be deduced with system identification. For that purpose, a wide range of methods is available, selection of the most suitable method for a given problem is a non-trivial matter.

Selected results obtained with the CFD/SI approach are reviewed, supplemented by *best practice* recommendations for successful and accurate identification of ROMs from time series data. Perspectives for the use of this method in other fields of science and engineering are developed.

1. Introduction

For the simulation of multi-physics, multi-scale phenomena, it is often advantageous to build a comprehensive system- or process-model from sub-models, which are purposely constructed to describe a certain aspect of the overall problem with high accuracy or efficiency. Such strategies of *divide et impera* ("divide and conquer") integrate modeling approaches of different complexity for different phenomena and scales.

The present author has extensive experience with thermo-acoustic combustion instabilities. This is a typical multi-physics/multi-scale phenomenon, involving fluid dynamics, mixing, combustion and heat transfer as well as generation, propagation, transmission and reflection of acoustic waves over a wide range of length and time scales (see Fig. 1). All relevant phenomena can be described within the framework of an unsteady, compressible, turbulent reacting flow model (possibly also including conjugate heat transfer). Thus, a corresponding computational fluid dynamics (CFD) formulation should *in principle* be sufficient to analyze or predict combustion instabilities. In the simplest case, one would start a transient simulation from a slightly perturbed initial condition and check for transient growth of unstable modes. However, such a straightforward, "brute force" application of numerical simulation to combustion instabilities faces the following challenges:

- Due to the wide range of length and time scales involved, huge computational resources are required.
- Mach numbers are typically low, such that the eigenvalues of the propagation operator, i.e. speed of sound and flow velocity, differ by at least an order of magnitude. This makes the calculation of compressible flow numerically difficult, especially if acoustic wave propagation must be modeled accurately.
- The formulation of appropriate boundary conditions, which represent the correct acoustic impedance (reflection coefficient) to outgoing acoustic waves, is difficult.
- Only the dominant unstable mode can be detected, a comprehensive modal analysis is not possible.

These issues are discussed further by Poinsot and Veynante (2005); Polifke et al. (2006); Selle et al. (2004); Kopitz and Polifke (2005); Worlikar et al. (1998).

* Tel.: +49 89 289 16216.
E-mail address: polifke@td.mw.tum.de
URL: http://www.td.mw.tum.de/tum-td/de/

0306-4549/$ - see front matter © 2013 Elsevier Ltd. All rights reserved.
http://dx.doi.org/10.1016/j.anucene.2013.10.037

A modeling strategy of *divide et impera* seems clearly advised for thermo-acoustic instabilities requires. Many methods developed for the analysis of thermo-acoustic combustion instabilities – see e.g. Keller, 1995; Bohn and Deuker, 1993; Dowling, 1995; Culick, 2006; Krebs et al., 1999; Pankiewitz and Sattelmayer, 2003; Benoit and Nicoud, 2005; Polifke, 2010a – can indeed be regarded as implementations of such a strategy. Typically, low-order or finite-element/-volume based models for acoustic wave propagation are combined with sub-models, that describe the response of the heat source to flow perturbations, or the acoustic impedance that upstream/downstream sections impose at the boundaries of the combustion chamber, say.

As part of such a strategy, CFD simulation can be combined with tools from system identification to characterize the response of a heat source to flow perturbations. For example, if a velocity-sensitive heat source like a hot wire in cross flow, or a premix flame is considered, one would determine its *frequency response* (also called *flame transfer function* in the particular case of a flame), which describes in a very compact way the linearized dynamics of the heat source:

$$F(\omega) \equiv \frac{\dot{Q}'(\omega)/\bar{\dot{Q}}}{u'_{\rm ref}(\omega)/\bar{u}_{\rm ref}}. \tag{1}$$

Here $\dot{Q}$ is the integral heat release rate of the flame, and $u_{\rm ref}$ a reference velocity at a location a short distance upstream of the heat source. The overbar $''\bar{\dots}''$ and the prime $''\dots'''$ indicate averages and fluctuations (or the Fourier transform thereof), respectively.

This review discusses a methodology, where first a time-dependent CFD simulation of the heat source with imposed broad-band excitation of the upstream velocity $u_{\rm ref}$ is performed. The resulting time series of fluctuating "signal" u' and "response" $\dot{Q}'$ (see Fig. 2) are then post-processed with methods from *system identification* (SI) to determine the frequency response $F(\omega)$. In this way a *reduced order model* of the flame dynamics in terms of the impulse response (see below) or the frequency response $F(\omega)$ is generated by the combined application of computational fluid dynamics and system identification (CFD/SI). The model reduction process is *data based*, as model coefficients are determined from time series data. One speaks of a *black box model*, because the reduced model is not determined by analytical derivation from the underlying governing equations of the problem. For subsequent stability analysis, the reduced model of the flame dynamics would be incorporated in a suitable thermo-acoustic system model.

Alternatively, if one is interested in the transmission, reflection, dissipation and amplification of acoustic waves at "acoustic elements" (discontinuous area changes, orifices, burners, etc.), one is interested to know the *scattering matrix* **S**,

$$\begin{pmatrix} f_d \\ g_u \end{pmatrix} \equiv \mathbf{S}(\omega) \cdot \begin{pmatrix} f_u \\ g_d \end{pmatrix}. \tag{2}$$

Here f and g represent the characteristic waves traveling in the down- and upstream direction, whereas the subscripts u and d refer to reference positions upstream and downstream of the element, respectively. For simple geometries, the scattering matrix of an element can be derived from the (linearized) equations of conservation of mass and momentum and suitable additional assumptions. However, in general the determination of the matrix coefficients from first principles is not possible, and one has to resort to experiment or numerical simulation. Experience with the experimental determination of transfer matrices has shown that this approach requires very careful experimental work – especially in the presence of turbulent flow or combustion – sophisticated post-processing, and long test runs, if the transfer matrix is to be recorded accurately over a range of frequencies (Åbom and Bodén, 1988; Paschereit and Polifke, 1998; Schuermans et al., 2004).

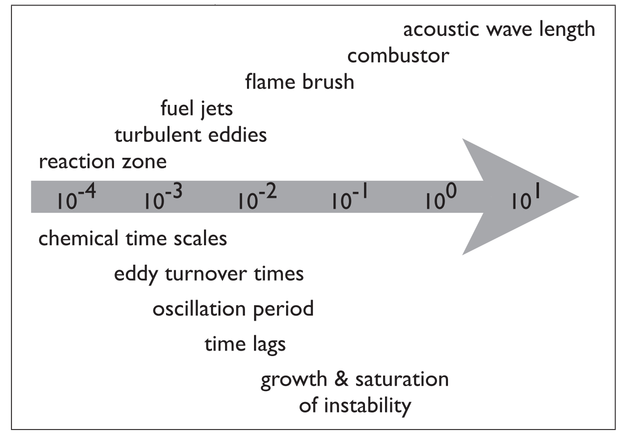

Fig. 1. Order of magnitude of relevant length (top, (m)) and time scales (bottom, (s)).

Again, it seems prudent to use SI to get the matrix coefficients from computational time series data generated with CFD, in particular *computational aero-acoustics* (CAA). In this context the incoming characteristic wave amplitudes f_u, g_d are considered as "signals", and the outgoing waves f_d, g_u as "responses", see again Fig. 2. Here the acoustic element is considered as a *multiple-input, multiple-output* (MIMO) element (whereas a velocity sensitive heat source would correspond to a *single-input, single-output* SISO element). Referring to Fig. 2, we note that with the methods presented here, both the frequency response $F(\omega)$ of the flame as well as the scattering matrix $\mathbf{S}(\omega)$ of burner and flame may be obtained with a single CFD run.

Compared to "brute force" CFD as discussed above, the CFD/SI approach does not require to model the complete combustion system, including air supply or flue gas exhaust, with possibly non-trivial acoustic boundary conditions. Instead, it should suffice to model the immediate vicinity of the heat source or acoustic element, and work with non-reflecting boundary conditions, resulting in significantly reduced computational effort. Furthermore, with *divide et impera* it should be possible to identify also weakly unstable or stable modes, and quickly assess the impact of acoustic boundary conditions (or addition of dampers like Helmholtz resonators) on system stability.

It has already been said that the CFD/SI approach can be seen as a data-based method to generated reduced order black-box models of (turbulent, reactive, compressible) flows. Note that the reduction in model order is indeed impressive: the data sets generated with Large Eddy Simulation of turbulent flames, say, correspond to $\mathcal{O}(10^9)$ degrees of freedom, while the frequency response of a flame can be represented with good resolution by a few hundred model coefficients. Results achieved with the CFD/Si method in the thermo- and aero-acoustic context are reviewed in the present article. In the next chapter, a compact introduction to system

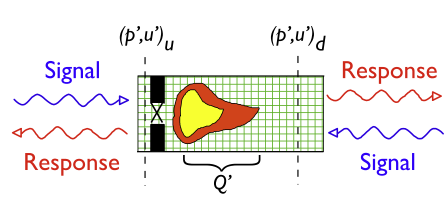

Fig. 2. Schematic of CFD/SI setup for identification of transfer function/matrix of a premix flame/burner. Overall rate of heat release $\dot{Q}'$ is obtained as a volume integral over heat release density. Fluctuations of pressure p' and velocity u' at sampling planes upstream (u) and downstream (d) of the burner are converted into characteristic wave amplitudes f, g and interpreted as "signals" and "responses".