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Review

Review and comparison of effective delayed neutron fraction calculation methods with Monte Carlo codes

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ABSTRACT

The calculation of the effective delayed neutron fraction, β_{eff}, with Monte Carlo codes is a complex task due to the requirement of properly considering the adjoint weighting of delayed neutrons. Nevertheless, several techniques have been proposed to circumvent this difficulty and obtain accurate Monte Carlo results for β_{eff} without the need of explicitly determining the adjoint flux. In this paper, we make a review of some of these techniques; namely we have analyzed two variants of what we call the k-eigenvalue technique and other techniques based on different interpretations of the physical meaning of the adjoint weighting. To test the validity of all these techniques we have implemented them with the MCNPX code and we have benchmarked them against a range of critical and subcritical systems for which either experimental or deterministic values of β_{eff} are available. Furthermore, several nuclear data libraries have been used in order to assess the impact of the uncertainty in nuclear data in the calculated value of β_{eff}.

Contents

1. Introduction

The effective delayed neutron fraction β_{eff} is a crucial parameter in reactor safety since it corresponds to the maximum reactivity that can be inserted in a critical system without becoming prompt-critical. This parameter is also fundamental to describe the kinetic and dynamic response of both critical and subcritical nuclear systems to internal or external perturbations.

Calculation methodologies for β_{eff} must take into account that it is an adjoint-weighted parameter. Since the calculation of adjoint fluxes with Monte Carlo codes is cumbersome, β_{eff} is usually calculated with deterministic codes. Nevertheless, its calculation with Monte Carlo codes is also desirable since they allow dealing with more complex geometries, different materials and continuous energy cross sections. The need of accurate calculation tools for β_{eff} is specially relevant in the case of ADS that cannot become critical, since the experimental determination of β_{eff} is usually very difficult in a subcritical state.

For this reason, a large number of publications have appeared over the last years considering different techniques for the calculation of β_{eff} with Monte Carlo codes. Trying to group them, we have classified them into two categories. The first one comprises techniques based on k-eigenvalue calculations; the second one comprises techniques based on different interpretations of the adjoint weighting, such as those based on interpreting the adjoint

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weighting as the next fission probability or the iterated fission probability. In addition, a third category of techniques can be considered to include those based on perturbative methods, such as these derived in Nagaya and Mori (2005, 2011), but we have not considered them in this paper.

In the first of these categories, techniques based on k-eigenvalue calculations, we include techniques based on the definition and calculation of certain parameters by analogy to the effective multiplication constant (k_{eff}), such as those of Bretscher (1997) and Spriggs (2001). Techniques based on the interpretation of the next-fission probability as the adjoint weighting have been analyzed by Nauchi and Kameyana (2005), Meulekamp and Van der Marck (2006) and Nagaya (2010). Techniques based on the interpretation of the iterated fission probability as the adjoint weighting can be seen as an improvement of the previous ones, and they have been proposed by Nauchi and Kameyana (2010), Raskach et al. (2010), Chiba (2011) and Irwanto et al. (2010). In Section 2 we will provide some discussion on the derivation and the physical meaning of all these techniques.

In Sections 3 and 4 we will present the results of the application of the above mentioned techniques against a number of critical and subcritical benchmark systems, that we consider to be representative of a wide range of nuclear systems. For this we have used the Monte Carlo code MCNPX[2] (Pelowitz et al., 2006) with three different nuclear data libraries (ENDF/B-VII.0, JEFF-3.1 and JENDL-3.3). The use of several nuclear data libraries allows us to set a lower limit to the uncertainty on β_{eff} estimators, due to both the accuracy of the different techniques and the uncertainties of the basic nuclear data.

2. Calculation methodologies

The usual definition of β_{eff} is:

$$\beta_{eff} = \frac{\left(\Phi_\lambda^\dagger, \hat{F}_d \Phi_\lambda\right)}{\left(\Phi_\lambda^\dagger, \hat{F} \Phi_\lambda\right)} \tag{1}$$

where $\hat{F}$ is the creation operator, that takes into account all neutrons (prompt and delayed) created in the phase space by fission, and $\hat{F}_d$ is the delayed neutron creation operator, that takes into account only delayed neutrons. The brackets indicate integration over the whole phase space. More specifically, the expressions in the numerator and in the denominator of Eq. (1) can be expanded as:

$$\left(\Phi_\lambda^\dagger, \hat{F}_d \Phi_\lambda\right) = \int \Phi_\lambda^\dagger\left(\vec{r}, E, \vec{\Omega}\right) \Sigma_f(\vec{r}, E') \nu_d(E') \times \chi_d\left(E', \vec{\Omega}' \to E, \vec{\Omega}\right) \Phi_\lambda\left(\vec{r}, E', \vec{\Omega}'\right) dE' d\vec{\Omega}' dE d\vec{\Omega}\, d\vec{r} \tag{2}$$

and

$$\left(\Phi_\lambda^\dagger, \hat{F} \Phi_\lambda\right) = \int \Phi_\lambda^\dagger\left(\vec{r}, E, \vec{\Omega}\right) \Sigma_f(\vec{r}, E') \nu(E') \times \chi\left(E', \vec{\Omega}' \to E, \vec{\Omega}\right) \Phi_\lambda\left(\vec{r}, E', \vec{\Omega}'\right) dE' d\vec{\Omega}' dE d\vec{\Omega}\, d\vec{r} \tag{3}$$

$\nu(E')$ and $\nu_d(E')$ denote, respectively, the average number of total (both prompt and delayed) and delayed neutrons at energy E' produced per fission. $\chi\left(E', \vec{\Omega}' \to E, \vec{\Omega}\right)$ and $\chi_d\left(E', \vec{\Omega}' \to E, \vec{\Omega}\right)$ represent, respectively, the spectrum of energy and angular distribution $\left(E, \vec{\Omega}\right)$ of the total and delayed neutrons produced by an incoming neutron with $\left(E', \vec{\Omega}'\right)$. Σ_f is the macroscopic fission cross section. Finally, Φ_λ and $\Phi_\lambda^\dagger$ are respectively the λ-mode direct and adjoint neutron fluxes, that is, the fundamental mode solutions of the eigenvalue equations:[3]

$$\hat{M} \Phi_\lambda = \frac{1}{k_{eff}} \hat{F} \Phi_\lambda \tag{4}$$

$$\hat{M}^\dagger \Phi_\lambda^\dagger = \frac{1}{k_{eff}} \hat{F}^\dagger \Phi_\lambda^\dagger \tag{5}$$

being $\hat{M}$ the migration and losses operator, that takes into account the net number of neutrons leaving the phase space element by capture, out-scattering or streaming, and $\hat{F}$ is the creation operator, already defined. $\hat{M}^\dagger$ and $\hat{F}^\dagger$ are their corresponding adjoints.

In this work, we consider only "effective" the delayed neutron fraction defined in Eq. (1) with the fluxes Φ_λ and $\Phi_\lambda^\dagger$. Several other delayed neutron fractions β can be defined considering fluxes other than Φ_λ or $\Phi_\lambda^\dagger$ but they will not be the "effective" values anymore. See, e.g., Bell and Glasstone (1970), Henry (1975) or Ott and Neuhold (1985) for further discussions on this topic. For instance, considering the adjoint flux to be constant over the whole phase space, we can define a non-adjoint weighted delayed neutron fraction, β_0, that can be expressed as:

$$\beta_0 = \frac{\left(\hat{F}_d \Phi_\lambda\right)}{\left(\hat{F} \Phi_\lambda\right)} \tag{6}$$

The determination of β_0 with Monte Carlo codes poses no major difficulty and can be performed by simply counting the number of total and delayed neutrons produced in fission processes. On the contrary, the determination of adjoint-weighted parameters requires the development of specific methodologies.

2.1. k-eigenvalue methods

Some of these methodologies can be classified as *k-eigenvalue methods* because they are based in defining and solving eigenvalue equations similar to (4) and (5). A first method is applied by Bretscher (1997) and it has been named the *prompt method* by Meulekamp and Van der Marck (2006) and the *prompt k-ratio method* by Nagaya and Mori (2011). It is obtained by defining the following eigenvalue equations:

$$\hat{M} \Phi_p = \frac{1}{k_p} \hat{F}_p \Phi_p \tag{7}$$

$$\hat{M}^\dagger \Phi_p^\dagger = \frac{1}{k_p} \hat{F}_p^\dagger \Phi_p^\dagger \tag{8}$$

where $\hat{F}_p$ is the prompt neutron creation operator. Assuming that $\Phi_p \simeq \Phi_\lambda$, which in principle seems a good approximation since over 99% of the neutrons produced in fission are prompt neutrons, we can obtain that:

$$\beta_{eff} = \frac{\left(\Phi_\lambda^\dagger, \hat{F}_d \Phi_\lambda\right)}{\left(\Phi_\lambda^\dagger, \hat{F} \Phi_\lambda\right)} = 1 - \frac{\left(\Phi_\lambda^\dagger, \hat{F}_p \Phi_\lambda\right)}{\left(\Phi_\lambda^\dagger, \hat{F} \Phi_\lambda\right)} \simeq - \frac{\left(\Phi_\lambda^\dagger, k_p \hat{M} \Phi_\lambda\right)}{\left(\Phi_\lambda^\dagger, k_{eff} \hat{M} \Phi_\lambda\right)} = 1 - \frac{k_p \left(\Phi_\lambda^\dagger, \hat{M} \Phi_\lambda\right)}{k_{eff} \left(\Phi_\lambda^\dagger, \hat{M} \Phi_\lambda\right)} = 1 - \frac{k_p}{k_{eff}} \tag{9}$$

[2] It must be remarked that the latest versions of the MCNP code can also provide values for adjoint-weighted parameters. The calculation methodology is based on an interpretation of the adjoint flux as iterated fission probability (Kiedrowski et al., 2010, 2011).

[3] Notice that the λ-mode flux, Φ_λ, obtained as solution of Eq. (4) only corresponds to the physical flux for a critical system. As the system departs from criticality, the physical flux also begins to differentiate from Φ_λ. Hence, the concept of effective delayed neutron fraction losses significance for systems far away from critical.