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# Contributions to the inverse problem of radiation transport

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ABSTRACT

We present an integral equation that describes the uncollided particle flux for the case of an inward spherical shell source of radius *R*. This is a reasonable description, for example, of a point source that moves on a spherical surface located at distance *R* from the target of a radiation treatment. The additional assumption of conditions for radial symmetry allows the derivation of an integral equation that relates the scalar flux to the description of the beam source as function of the angle between the direction of the source particles and the normal to the sphere.

Analytical and numerical solutions for this integral equation are successfully compared with, respectively, known analytical results and with Monte Carlo simulations. The integral equation can then be used for solutions of the inverse problem: given the flux obtain the source, i.e. the shape of the beam. A numerical algorithm was developed for this purpose as well as an analytical solutions based on the solution of the integral equation by the use of the Laplace transform.

The optimal shape for the beam is then obtained based on the constraint that the source has to be positive and finite everywhere, allowing the design of appropriate collimators for the beams. Monte Carlo calculations as a function of the number of collisions show that the uncollided flux for the beam so determined behaves as expected and that penumbra effects due to multiple collisions are sufficiently small (∼20%) to consider the beam as a good first guess for an iterative procedure for the design, for example, of 3-D conformal radiotherapy treatment.



## 1. Introduction

A typical problem related to the transport of nuclear particles for nuclear engineering applications is the direct problem: given the sources obtain the particles fluxes and their functionals. Many algorithms for computer calculations (RSICC, 2013) were developed and they are available for the design of a variety of systems from nuclear reactors to shielding requirements.

The inverse problem in transport theory, i.e. given the fluxes and their functionals, compute the sources, is obviously much more complex and difficult to solve but it is of absolute importance in fields like radiotherapy or in non destructive assay methods to detect the presence of unusual conditions for the tested material.

Given the problem of the optimization of the particles fluxes in an extended region, available algorithms and codes for the direct method allow two oppositive approaches: (1) to analyze the sensitivity of the response in the extended region to changes in one point of the phase space for the source (trial and error) or (2) to analyze the sensitivity of the response in one point of the extended region to all possible changes of the source (Difilippo, 1998). Ideally the solution of the inverse problem would overcome these limitations.

Presently, three dimensional conformal radiotherapy planning for cancer patients involves the optimization of the radiation field through two steps: (1) the calculations of the sensitivities of the dose to the description of the source in phase space, and (2) the use of the sensitivities in an iterative optimization algorithm to define the source. Any good guess to start the iterations might substantially reduce the number of iterations required to solve the problem. The goal of this work is to help to find a good guess for the beam source.

Our idealized inward source is located at the surface of a sphere of radius *R*, which is a reasonable description for real treatments or assay methods. Section 2 is dedicated to the free streaming, i.e. there is nothing within the sphere, the integral equation for the flux is solved for the direct and inverse problems. Analytical and numerical solutions are then compared with Monte Carlo simulations.

Section 3 deals with the direct and inverse problem for the uncollided flux in a media inside the sphere. Materials, concentrations and dimensions correspond to the human body. Results are verified through Monte Carlo simulations to show that the calculated beams are a good first guess for radiotherapy planning. Applications to the design of collimators for photon sources are shown in Section 2.

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## 2. Radiation field inside an empty spherical shell source

In this section we write the general equations for the case of an inward spherical shell source, further assumptions of conditions for radial symmetry allow the derivation of a simple integral equation for the scalar flux, which is solved for the direct and inverse cases and corroborated with analytical and numerical tests. Some of the results for the direct problem were published by Case et al. (1953), we include them here for completeness.

Our inward shell source is located over a sphere of center O and radius $R$ as described in Fig. 1.

One general source point A, located on the sphere at angular coordinates ($\alpha$,$\beta$), produces particles in the direction $\vec{\Omega}(\theta_s, \psi_s)$, where the directional angles ($\theta_s$, $\psi_s$)are defined with respect to the normal to the sphere at point A. These particles move toward a field point B of coordinate $\vec{r}$ whose local flux direction is also $\vec{\Omega}(\theta, \psi)$ but referred to directional angles $(\theta, \psi)$ defined with respect to the direction of the field point $\vec{r}$ .

From the point of view of field point B, the contributions to the vector field at ($\mathbf{r}$, $\vec{\Omega}$) come from a point A whose angular coordinates are functions of $\mathbf{r}$ and $\vec{\Omega}$, i.e. ($\alpha = \alpha\vec{r}, \vec{\Omega}$) and $\beta = \beta(\vec{r}, \vec{\Omega})$. Similarly, the directional angles with respect to the normal to the sphere are $\theta_S = \theta_S(\vec{r}, \vec{\Omega})$ and $\psi_S = \psi_S(\vec{r}, \vec{\Omega})$.

Fig. 2 which corresponds to the slice of the sphere produced by plane OAB shows that the relationship between the polar angle of the source particle, $\theta_s$, and the polar angle of the field, $\theta$, is given by

$$\mu_s = \sqrt{1 - \rho^2 + \rho^2\mu^2} \tag{1}$$

where the polar angle for the source particles are measured with respect to the inward normal to the sphere, $\mu = \cos\theta$, $\mu_s = \cos\theta_s$ and $\rho = r/R$.

The vector field is then

$$F(\vec{r}, \vec{\Omega}) = S(\theta_s, \psi_s, \alpha, \beta)/\mu_S \tag{2}$$

the denominator in Eq. (2) corresponds to the ratio of field to source area element. It appears because the intensity of the source is defined per unit area of the sphere.

Considerable simplifications can be obtained as we assume conditions for radial symmetry, that is the intensity of the shell source is independent of the position over the sphere ($\alpha$, $\beta$) and independent of the azimuth angle $\psi_s$. Under these circumstances and with the source normalized to 1 particle for the whole sphere, $S(\mu_S, \psi_S) = (1/4\pi R^2)(1/2\pi)D(\mu_S)$, the vector flux integrated over azimuth is given by

$$f(r, \mu) = \int_0^{2\pi} F(\vec{r}, \vec{\Omega})d\psi = D(\mu_s(\mu))/(\mu_S(\mu)4\pi R^2) \tag{3}$$

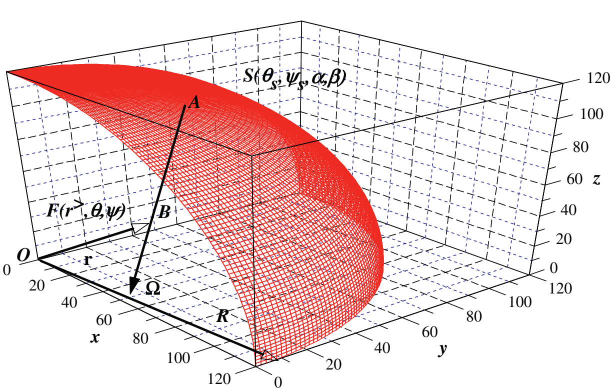


**Fig. 1.** Geometry for the analysis of the transport of particles produced in a spherical shell source of radius $R$ defining source and field points, A and B, respectively.

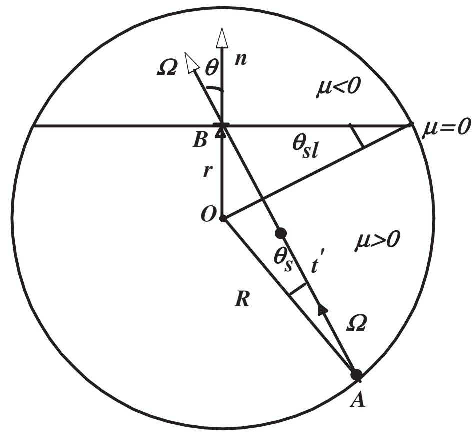


**Fig. 2.** Cut OAB of Fig. 1 defining polar angles for the source and the field points.

where $D(\mu_S)$, the source per unit $\mu_S$, is normalized to one incoming particle $\int_0^1 D(\mu_S)d\mu_S = 1$.

The vector flux given by Eq. (3) can now be integrated on the polar angle to obtain the scalar flux. Fig. 2 shows two contributions: one coming from "below", for positive $\mu$ and one coming from "above", for negative $\mu$. Eq. (1) allows us to change the integration on $\mu$ to an integration in $\mu_s$, in this way the contribution from $\mu > 0$ is

$$\Phi_+(\rho) \equiv \int_0^1 f(r, \mu)d\mu = \frac{1}{4\pi R^2\rho}\int_{\sqrt{1-\rho^2}}^1 \frac{D(\mu_s)}{\sqrt{\rho^2 - 1 + \mu_s^2}}d\mu_s \tag{4}$$

and the contribution from $\mu < 0$ is

$$\Phi_-(\rho) \equiv \int_{-1}^0 f(r, \mu)d\mu = \frac{1}{4\pi R^2}\int_{\sqrt{1-\rho^2}}^1 \frac{D(\mu_s)}{\sqrt{\rho^2 - 1 + \mu_s^2}}d\mu_s \tag{5}$$

i.e. equal to $\Phi_+(\rho)$ because in Eq. (5) $\mu = -\sqrt{1 + (\mu_s^2 - 1)/\rho^2}$.

The total scalar flux is then given by the integral equation

$$\Phi(\rho) = \frac{1}{2\pi R^2\rho}\int_{\sqrt{1-\rho^2}}^1 \frac{D(\mu_s)}{\sqrt{\rho^2 - 1 + \mu_s^2}}d\mu_s \tag{6}$$

The next two sections are dedicated to the direct and inverse problem defined by this equation. Cross checking between analytical and numerical results associated with the integral equation are made and additionally they are compared with direct calculation with the Monte Carlo code MCNP (X-5 Monte Carlo Team, 2003).

### 2.1. Analytical and numerical calculations for the direct problem in an empty shell

We develop here an algorithm to solve numerically the integral Eq. (6) which can work in both directions, for the direct and the inverse problems. We discussed also some analytical results which can be used to test the algorithm; additionally comparisons with MCNP direct calculations give additional support to the methodology developed.

#### 2.1.1. Some analytical results for the direct problem in an empty shell

Some simple source distributions for the case of free streaming produce analytical solutions for the flux. By direct application of Eq. (3), for the vector flux, and direct integration in $\mu$, we obtain the results of Table 1 for the cases of isotropic, lineal, quadratic and beam dependence for the intensity of the source as function of $\mu_s$. The first three cases were already obtained by Case et al. (1953).