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A space-averaged model for hollow fibre membranes filters☆

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ABSTRACT

We present a space independent model to describe the filtration process taking place in a hollow fibre membrane filter. This method is based on averaging a full three-dimensional model made by a system of PDEs describing the hydrodynamic process coupled with transport, adsorption and attachment equations. Few crucial assumptions are needed to guarantee the physical coherence of the model, but a practical way to check the fulfilment of these constraints is provided, so that one can quickly verify the applicability of the model. This work has been tested in the framework of a collaboration project with a manufacturing company to provide an optimization of both product and process. To this purpose, a program was written in *Python* programming language, implementing both the calibration and the forward simulation part, in order to be used by the membranes manufactures for obtaining information on the filter efficiency.

1. Introduction

The use of cylindrical hollow porous fibres in filtering devices is a purification technology widely used in many different applications.

In particular we consider the filtration modules employed in water filtration processes. This device consists in a cylindrical pressure vessel housing a bundle of hollow-fibre membranes having a pores diameter in the range 0.01–0.1 μm: namely, we consider the so-called *ultrafiltration* application (Lorain, Espenan, & Hugaboom, 2003). The *feed* (i.e. the water to be filtered) goes into the module from an edge and flows in the space outside the fibres: a pressure gradient is applied between the outer part of the fibre and the inner one, so that the water flow through the membranes and all the particles larger than the pores diameter are cut off. The purified water (named *permeate*) flows away by the outlet placed at the opposite edge of the inlet. This configuration is called *dead-end filtration* (because all the entering water is forced to flow through the membrane) and it is an alternative to the *cross flow filtration* (see Belfort, Davisb, & Zydney, 1994 for more details). In this filtering technology the main problem is that the pressure gradient necessary to maintain a constant flux of filtrate increases due to various forms of *fouling* phenomena, namely the process of membrane soiling (Lorain et al., 2007). Indeed, a part of the filtered particles can attach on the outer surface of the membrane, forming a thin layer (the so called *cake*) which eventually coats all the medium and reduces the filtration efficiency (Broeckmann, Busch, Wintgens, & Marquardt, 2006). To remove such a material, periodically a backwash process is imposed to the system, inverting the flux and let the clean water flow through the membrane. In addition, also air scouring and chemical cleaning can be applied during the backwash stage. Another type of fouling is the one given by the adsorption of the pollutant inside the membrane matrix. Basically, this process reduces porosity and permeability of the membrane and it cannot be removed by any purely mechanical action. For this reason it is called *irreversible fouling*. A single cycle of production takes almost 1 h; afterwards a backwash cycle (almost 60 s) is imposed, along with an air scouring in order to make easier the removal of the cake from the membrane.

A direct and detailed experimental study of the fouling evolution is rather involved. The main difficulty is that a measurement of changes occurring near the membrane surface may significantly affect the actual process of filtration: therefore, an investigation during real operating conditions seems to be very hard, or even impossible.

To partially cover this lack, during last two decades several models to simulate the filtration based on hollow fibre membrane have been presented: in Belfort et al. (1994) and Bowen and Janner (1995) the reader can found a good review of the different approaches. One of the main difference lies on the scale at which the problem is addressed. An example of the micro-scale approach can be found in Borsi, Farina, and Fasano (2011), Carroll and Booker (2000), Kelsey, Pillarella, and Zydney (1990), Kostoglou and Karabelas (2008) and the references therein. Moreover, Kumar

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Nomenclature

Nomenclature

$\mathbf{q}$	specific discharge (m/s)
$\mathbb{K}$	permeability tensor (m^2)
k	permeability (m^2)
N	number of fibres
N_p	number of pores
r_o	fibre outer radius (m)
r_i	fibre inner radius (m)
l_m	membrane thickness (m)
R	module radius (m)
H	module length (m)
P	pressure (kPa)
P_{in}	inlet pressure (kPa)
P_{out}	outlet pressure (kPa)
TMP	trans-membrane pressure (kPa)
g	gravity acceleration (m s^{-2})
A	area of the module section (m^2)
A_{filt}	filtering area (m^2)
A_v	specific filtering area (m^{-1})
R_m	membrane resistance (m^{-1})
R_c	cake resistance (m^{-1})
c	pollutant concentration in the volume of water (kg/m^3)
c_m	cake concentration (kg/m^3)
c_p	mass fraction of adsorbed pollutant
K_F	Freundlich adsorption constant (m^3/kg)
D_s	hydrodynamic dispersion coefficient (m^2/s)
d	membrane pores diameter (m)
d_0	membrane pores diameter (in a clean membrane) (m)
d	membrane pores diameter (m)
Q_{in}	feed flow (l/h)
J_{in}	feed flux (m/s)
Q_{back}	feed flow during backwash (l/h)
J_{back}	feed flux during backwash (m/s)
T_{filt}	filtration time (s)
T_{back}	backwash time (s)
N_{cycles}	number of filtration/backwash stages

Greek letters

Γ	source/sink term between lumina and shell (s^{-1})
μ	viscosity (Pa s)
ρ	density (kg/m^3)
ε	porosity
α	attachment coefficient
γ_c	cake permeability (m^2)
η	adsorption constant (m^{-2})
δ	cake thickness (m)

Subscripts

s	shell region
l	lumina region
m	membrane
c	cake

and Upadhyay (2000) give a useful summary of the different correlations between the velocity profile and the pressure distribution near a permeable wall.

Conversely, in the present study we refer to a macro-scale description of the process taking place in a whole filtering module. An accurate description requires the definition of a system of PDEs accounting for the full 3D problem, even if simplifications due to the geometrical symmetry can be applied. On the other hand, the numerical solution of these models requires a huge time when dealing with long-term simulations corresponding to several days of production.

An alternative method consists in defining a relationship between trans-membrane pressure (TMP) and the *membrane resistance*, depending only on time. Such a resistance depends on the fouling process. The models belonging to this class are known as *resistance in series models* (Belfort et al., 1994; Katsoufidou, Yiantsios, & Karabelas, 2005) and they give an averaged description of the process. Because of their reduced complexity, their solution is quick and simple. Nevertheless, very often in these solutions the direct dependency on the module geometry and the membrane structure is lost: in general many unknown parameters are present and an accurate calibration has to be run in order to link this general formulation with the specific device considered. In the present work we define a "zero-dimensional" model (namely spatial independent) which is the result of an average procedure made on a quasi-3D model. Even if the fulfilment of some important assumption has to be checked in order to guarantee the physical coherence of the model, this approach is a good compromise between the accuracy of a full 3D approach and the standard resistance-in-series methods.

The paper is organized as follows. In Section 2 the general model is reported, along the definition of the physical quantities and the notation used through the paper. In Section 3 the crucial physical assumptions are reported and the averaging procedure based on that is described. The practical formulas to simulate the filtration process are then presented. Finally, in Section 4 a comparison with experimental data is reported in order to evaluate the validity of the model.

2. The quasi-3D model

In order to describe the filtration process, we consider the module as a double porosity and double permeability porous medium. This method is widely used in the macroscopic description of flow in porous media presenting two or more different porosities. A typical example is the modelling of fractured media, where porous matrix and fractures are treated as two interconnected porous media (Barenblatt, Entov, & Ryzhik, 1990). Even if we are dealing with a completely different application, here we use the same conceptualization. Therefore, let us define two regions:

- the *lumina region*, namely a porous cylinder equivalent to the module, where the void space consists in the total volume occupied by the inner part of the fibres. In other words, we consider a *capillary tubes* structure.
- the *shell region*, namely a porous cylinder where the void space is the space outside the fibres, while the solid matrix is the whole volume occupied by the fibres.

Therefore, in these two different regions the void spaces are somehow complementary and the total volume occupied by the membrane itself is a sort of "interface" between these regions. For the reader convenience, in Fig. 1 we report a schematic picture showing the two different regions, and the membrane acting as an interface. As for the hydrodynamic, the two regions are linked each other by a source/sink term representing the water flux through the membrane. A similar approach can be found in Labecki, Piret, and Bowen (1995). In particular, we notice that this method assumes a uniform distribution of the fibres inside the module, which is a reasonable assumption in our case. Efforts on modelling the effects of a non-uniform distribution can be found in (Zheng, Xu, & Xu, 2003).