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Performing uncertainty analysis of a nonlinear Point-Kinetics/Lumped Parameters problem using Polynomial Chaos techniques

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ABSTRACT

Uncertainty analysis methodologies represent an important tool in the field of reactor physics with applications which span from the design phase to the safety analysis, as a support to "best estimate" models. A major source of uncertainty in reactor simulations is the input data set of the problem which is propagated, throughout the model, to the final simulation output. In this paper we perform such a propagation for a nonlinear point-kinetic model coupled to a lumped parameters system using a spectral technique, based on the Polynomial Chaos Expansion (PCE). We present two different ways to implement this technique, together with an overview of standard methods, and we apply them to a positive reactivity insertion transient. We show that for low-dimensional coupled problems PCE methods achieve the precision of Monte Carlo approaches at a significantly reduced computational cost.

1. Introduction

The role of uncertainty analysis in nuclear reactor simulations is becoming increasingly important and challenging as their size and precision is constantly increasing. Moreover, regulations used in reactor licensing have begun to allow the "best estimate plus uncertainty" simulations approach, increasing the need for reliable and precise uncertainty analysis methodologies (NEA, 2008). There are different kinds of uncertainties to deal with, some of which are represented by approximations introduced by the model or by the numerical scheme used to solve it, however, one of the most important challenges in Uncertainty Analysis (UA) is to handle the uncertainty present in the input data of the problem (like the material properties or the geometric descriptions).

This corresponds to estimating how the lack of knowledge in the input data set influences the simulation outputs used in design and safety analysis. Many techniques have been implemented and used in the field so far, the main methods being statistical and deterministic. The main distinction between the two is that statistical methods are exact and require a large computational effort while deterministic methods rely on model approximations which make the technique faster compared to the first approach. A common way to propagate uncertainties using a deterministic approach is the application of first order perturbation techniques based on adjoint formulations. The computational effort required to perform this propagation is relatively small and the accuracy in the prediction of the output uncertainty for small perturbations is good even for nonlinear problems (Cacuci, 2003).

Unfortunately within a safety analysis framework we are in presence of strong nonlinearities, and large (possibly) non-Gaussian input uncertainties often in range where the linear approximation introduced by perturbation methods would not hold anymore. In this paper we present the application of spectral techniques, based on the Polynomial Chaos Expansion introduced by Wiener (1938), to a coupled time-dependent model in order to propose an alternative to standard methodologies. PCE based techniques were first proposed by Ghanem and Spanos (1991) and have been applied so far to different scientific fields, ranging from Computational Fluid Dynamics problems (Najm, 2009; Mathelin et al., 2005) to structural mechanics (Ghanem and Spanos, 1997).

Two main PCE approaches, categorized as intrusive and non-intrusive, can be used to implement these spectral techniques. As the name suggests the main difference between the approaches is that with the former it is possible to use the original code as a "black box" while the latter involves the definition of a newly coupled problem which needs to be coded and solved. Within the reactor physics field the application of an intrusive PCE approach was first presented for a neutron diffusion problem by Williams (2007) and later applied to the transport equation in two studies (Williams, 2006; Eaton and Williams, 2010) for fixed source and eigenvalue problems. This concept has been also extended to spatially random problems and to non-intrusive methods by Fichtl (2009) while Roderick et al. (2010) presented the application of a

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PCE based regression technique to a coupled steady-state problem. Regarding time-dependent problems the only application in the nuclear field was proposed by Hagues et al. (2010) where an intrusive stochastic method is applied to a radionuclide dispersion model. No application to time-dependent nonlinear problems has been presented so far.

In the present work we apply two PCE techniques, the Non-intrusive Spectral Projection (NISP) and the Stochastic Galerkin (SG) formulation, to a coupled time-dependent problem described using a Point-Kinetics/Lumped-Parameters model. The implementation and the performance of the two techniques are discussed and compared to standard methodologies. The paper is structured as follows: first the model used to simplify the coupled physical problem is introduced, then an overview of the traditional UA techniques, together with an introduction to the two main methods to implement the PCE are presented. In the final part the results of the application of these techniques to a coupled time-dependent problem, describing a reactivity insertion transient, are discussed and compared.

2. Derivation of the coupled time-dependent problem

The problem considered for the application of the Uncertainty Quantification techniques introduced in the previous section is described by a system of a coupled Ordinary Differential Equations (ODE) modeling the time-dependent behavior of a simplified reactor. The model is derived using a point-kinetic approximation for the neutron population (Duderstadt and Hamilton, 1976) together with a lumped parameter description of the reactor temperatures. These assumptions allow the elimination of the spatial dependencies and therefore focus on the time-dependent part. The point-kinetic system is

$$\begin{aligned}\frac{dP}{dt} &= \frac{\rho(T_f, T_c, t) - \beta}{\Lambda} P + \sum_{k=1}^{6} \lambda_k C_k \\ \frac{dC_k}{dt} &= -\lambda_k C_k + \frac{\beta_k}{\Lambda} P\end{aligned} \tag{1}$$

where P is the reactor power, Λ the mean generation time, C_k the concentration of the kth precursor group, β_k and λ_k the delayed neutrons fraction and the decay constant for the kth precursor group and β the total delayed neutrons fraction. The thermo-kinetics/thermal–hydraulics equations, needed to describe the removal of the heat by the coolant, are approximated using a lumped parameter model, i.e. averaging the unknown values over the whole domain. Assuming the reactor is divided into a fuel and a coolant region, their time-dependent average temperatures are described by the equations

$$\begin{aligned} M_f c_{pf} \frac{dT_f}{dt} &= P + Ah(T_c - T_f) \\ M_c c_{pc} \left[\frac{dT_c}{dt} + v \frac{T_c - T_{in}}{L} \right] &= Ah(T_f - T_c) \end{aligned}$$

where M_f and M_c are the fuel and coolant mass respectively, h the heat transfer coefficient, A the heat transfer surface, v the coolant flow velocity, L the channel length and T_{in} the inlet temperature of the coolant. The coupling between these two equations is given by the presence of the power production term P and by the time-dependent reactivity $\rho(t)$ in the point kinetic equation, defined as the contribution of three different terms

$$\rho(t) = \rho_{ext} + \alpha_D [T_f - T_f(0)] + \alpha_c [T_c - T_c(0)]$$

where ρ_{ext} represents an external reactivity insertion, α_D and α_c are the Doppler and the coolant reactivity coefficients respectively, and $T_f(0)$ and $T_c(0)$ are the initial system temperatures. We assume that the system starts from the following initial conditions

$$\begin{aligned} P(0) &= P_0 \\ C_k(0) &= \frac{\beta_k}{\lambda_k \Lambda} P_0 \\ T_f(0) &= T_c(0) + \frac{P_0}{Ah} \\ T_c(0) &= T_{in} + \frac{P_0 L}{M_c c_{pc} v} \end{aligned}$$

In order to present the different techniques that can be used to perform UA of the system hitherto discussed, it is useful to reformulate it using a more generic formulation. We reformulate the model as a generic system of ODEs defined as

$$\begin{aligned} \frac{d\mathbf{u}}{dt} &= \mathrm{L}(\boldsymbol{\alpha}, \mathbf{u}) \\ \mathbf{u}(0) &= \mathbf{U}_0 \end{aligned} \tag{2}$$

where L is a nonlinear operator, $\mathbf{u}$ the unknown solution of the problem, and $\boldsymbol{\alpha}$ the set of input parameters. In general, one may not only be interested in the solution of the previous system but also in a response R ($\boldsymbol{\alpha}$,$\mathbf{u}$,t) which can be described as a functional of the solution and the input parameter set. The first step required to propagate uncertainties throughout the model is the introduction of a random space $\boldsymbol{\theta} = [\theta_1, \ldots, \theta_N]$ which can be used to describe the stochastic component of the input parameter set, $\theta_1, \ldots, \theta_N$ are independent random variables used to model the random input data $\boldsymbol{\alpha}(\boldsymbol{\theta})$. The introduction of this uncertainty in the input data set turns the deterministic output of the model (represented by the unknown system parameters and by the response) into a stochastic one, with the output quantities $\mathbf{u}(\boldsymbol{\theta})$ and R ($\boldsymbol{\theta}$) depending on the same random variable set.

The propagation of uncertainties involves the determination of this dependency of the system outputs on the random space $\boldsymbol{\theta}$. Many methodologies can be applied to perform the task, in the next section a brief description of the two main approaches, Monte Carlo and Sensitivity Analysis, is given together with the introduction of the two main Polynomial Chaos Expansion techniques.

2.1. Standard uncertainty analysis methodologies

In this section we briefly describe the two main methodologies used in the reactor physics domain to perform UA, in order to compare them to the alternative spectral methods, which are as follows.

2.1.1. Monte Carlo (MC) methods

The main concept behind the Monte Carlo approach is quite straightforward: the random input data set is sampled M times until the statistical moments of the simulation output converge. The unbiased definition for the mean of a response R is

$$\mathrm{E}(\mathrm{R}) = \frac{1}{M} \sum_{i=1}^{M} \mathrm{R}(\boldsymbol{\theta}^i)$$

where $\boldsymbol{\theta}^i$ is a single realization of the random input set. The unbiased variance is expressed by the equation

$$\sigma^2(\mathrm{R}) = \frac{1}{M-1} \sum_{i=1}^{M} \left(\mathrm{R}(\boldsymbol{\theta}^i) - \mathrm{E}(\mathrm{R}) \right)^2$$

the statistical error associated with these moments is proportional to $(1/M)^{1/2}$. MC methods are relatively easy to implement and they do not require any modification of the original code used to calculate the output quantities. The use of the original mathematical model to calculate the statistical moments allows the consideration of physical phenomena that would be neglected using approximated