Annals of Nuclear Energy 37 (2010) 1595–1600

Contents lists available at ScienceDirect

Annals of Nuclear Energy

journal homepage: www.elsevier.com/locate/anucene

A response-based time-dependent neutron transport method

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ARTICLE INFO

Article history:
Received 26 November 2009
Accepted 18 March 2010
Available online 14 July 2010

Keywords:
Time-dependent transport theory
Response function
Reactor physics

ABSTRACT

An efficient response-based solution to the time-dependent neutron transport equation in a semi-infinite slab is derived. The solution is based on polynomial expansions of the source terms and neutron flux in the time domain. The expansion coefficients of the flux solution are computed in terms of response functions, which are special cases of Green's functions for arbitrary in-volume and surface sources. The resulting response equation, which is a convolution integral equation in time, is reduced to a linear algebraic system of equations in the expansion coefficients. Two example problems are solved using the response-based method, and the extension of the method to general (finite, heterogeneous) problems is discussed.

1. Introduction

Time-dependent neutron transport theory has evolved more slowly than its steady-state counterpart. This is partially because of the additional complexity incurred by adding the time dimension to the space, angle and energy variables. Additionally, realistic treatment of neutron transport in reactor systems necessitates including delayed neutrons that introduce multiple time scales to the transport solution. As a result, most modern transient analysis tools rely on approximations that simplify either the reactor physics or geometry. Two of the most widely adopted approaches are the point kinetics model (including the adiabatic and quasistatic approximations) (Ott and Meneley, 1969; Bell and Glasstone, 1979; Goluoglu and Dodds, 2001; Dulla et al., 2008) and nodal diffusion theory (Lawrence and Dorning, 1979; Alchalabi et al., 1993; Sutton and Aviles, 1996; Downar et al., 2004). The former approximates the transport solution by limiting the spatial variation of the flux distribution while the latter simplifies the reactor geometry by homogenizing large subregions of the reactor volume. As computing power has increased there has also been a trend towards a direct solution of the transport equation by traditional space–time discretization methods (Hill, 1976; Oliveira and Goddard, 1996; Pautz and Birkhofer, 2003). This approach, however, requires significant computational speed and memory to be feasible for large reactor systems. This paper introduces a novel time-dependent methodology for neutron transport for reactor physics applications that is both efficient and accurate.

The methodology employs a generalized approach based on response functions (Forget and Rahnema, 2006a; Mosher and Rahnema, 2006). Response functions characterize the time-dependent neutronic response of a system subjected to arbitrary incident and in-volume source terms. A response, in this context, can be any functional of the neutron angular flux, although quantities such as the exiting partial current, fission rate or the flux distribution itself tend to be the most useful. In this work, the responses and source terms are expanded in Legendre polynomial series in time. The response functions, which embody the relationship between source and response, provide the coupling needed to calculate the coefficients of the response expansion in terms of the source expansion coefficients. Once the response functions have been determined, the transport problem is reduced to the computation of a relatively small number of response coefficients.

Previous work has successfully implemented a response-function-based approach for full-core 3D steady-state reactor problems, showing both a high level of accuracy and significant improvements in efficiency relative to alternative methods (Forget and Rahnema, 2006b). In steady-state, response functions express the relationship between responses (the eigenvalue and fission rates) and sources (incoming partial currents) in the space, angle and energy domains. The current work extends the theory by additionally addressing the time-dependence of responses. Since this work represents a preliminary investigation into time-dependent response function theory, we will intentionally restrict our considerations to systems in which the space and angle dependence can be eliminated (i.e. semi-infinite slab geometries with uniform sources). This restriction is for convenience and simplicity; it is not a constraint imposed by underlying theoretical assumptions.

The following section presents a derivation of the time-dependent response equations. Section 3 presents some example problems and results, followed by some closing remarks and directions for future work.

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0306-4549/$ - see front matter © 2010 Published by Elsevier Ltd.
doi:10.1016/j.anucene.2010.03.010

2. Theory

Consider a semi-infinite fissile slab. The monoenergetic Boltzmann transport equation for neutrons propagating through such a region is

$$\frac{1}{v}\frac{\partial\psi(z,\mu,t)}{\partial t}+\mu\frac{\partial\psi(z,\mu,t)}{\partial z}+\sigma(t)\psi(z,\mu,t) = \sigma_s(z,t)\int_{-1}^{1}f(\mu'\to\mu,t)\psi(z,\mu',t)d\mu',\quad z,t\geqslant 0 \tag{1}$$

where μ is the cosine of the angle of neutron flight with respect to the slab axis. A uniform, isotropic initial condition is prescribed at $t = 0$ and an incident isotropic flux boundary condition is assigned to the free surface at $z = 0$:

$$\psi(0,\mu,t)=\Gamma_0(t),\quad \mu,t\geqslant 0 \tag{2}$$

$$\psi(z,\mu,0)=\frac{1}{2}\varphi_0,\quad z\geqslant 0 \tag{3}$$

The goal is to formulate Eq. (1) in a way that is amenable to calculating time-dependent responses caused by arbitrary boundary and initial conditions. To this end, we write both boundary and initial conditions explicitly as source terms in the transport equation so that Eqs. (1)–(3) become

$$\frac{1}{v}\frac{\partial\psi(z,\mu,t)}{\partial t}+\mu\frac{\partial\psi(z,\mu,t)}{\partial z}\sigma(z,t)\psi(z,\mu,t)=\sigma_s(z,t)\int_{-1}^{1}f(z,\mu'\to\mu,t)\psi(z,\mu',t)d\mu'+\Gamma_0(t)\delta(z)+\frac{1}{2}Q_0\delta(t),\quad t\geqslant 0 \tag{4}$$

where $Q_0=\frac{1}{v}\varphi_0$ (see Appendix A). The solution of this inhomogeneous equation may be expressed in terms of the Green's function, $G(z,\mu,t;z',t')$:

$$\psi(z,\mu,t)=\int_0^t dt'\Gamma_0(t')G(z,\mu,t;0,t')+\frac{1}{2}Q_0\int_0^\infty dz'G(z,\mu,t;z',0). \tag{5}$$

where the Green's function satisfies the equation:

$$\frac{1}{v}\frac{\partial G(z,\mu,t;z',t')}{\partial t}+\mu\frac{\partial G(z,\mu,t;z',t')}{\partial z}\sigma(z,t)G(z,\mu,t;z',t') = \sigma_s(z,t)\int_{-1}^{1}f(z,\mu'\to\mu,t)G(z,\mu',t;z',t')d\mu' +\frac{1}{2}\delta(z-z')\delta(t-t'),\quad t,t'\geqslant 0 \tag{6}$$

If the Green's equation is autonomous in time then

$$G(z,\mu,t-t';z',0)=G(z,\mu,t;z',t'). \tag{7}$$

Using this property, the variable of time integration in Eq. (5) can be changed from the absolute time t' to a relative response time, $\tau\equiv t-t'$. This substitution results in

$$\psi(z,\mu,t)=\int_0^t d\tau\Gamma_0(t-\tau)G(z,\mu,\tau;0,0)+\frac{1}{2}Q_0\int_0^\infty dz'G(z,\mu,t;z',0). \tag{8}$$

The first term on the right-hand-side of Eq. (8) represents the system response to the generic surface flux, $\Gamma_0(t)$, while the second term represents the system response to a uniform isotropic source of intensity Q_0. Both of these responses are special cases of the general Green's function, so we will consequently define the following surface and volume response functions, respectively:

$$R_S(z,\mu,\tau)\equiv G(z,\mu,\tau;0,0) \tag{9}$$

$$R_V(z,\mu,t)\equiv\int_0^\infty dz'G(z,\mu,t;z',0) \tag{10}$$

These response functions are independent of the boundary and initial conditions: the first is the flux solution resulting from a unit isotropic source pulsed at $t=0$ while the second is the flux solution resulting from a uniform isotropic source also pulsed at time $t=0$. One may therefore construct solutions to myriad source configurations (including various albedo conditions as a subset) using only the two response functions given above. This generality enables the efficiency of the current method. The governing response equation that forms the basis for this work is obtained by inserting the response function definitions into Eq. (8),

$$\psi(z,\mu,t)=\int_0^t d\tau\Gamma_0(t-\tau)R_S(z,\mu,\tau)+\frac{1}{2}Q_0R_V(z,\mu,t). \tag{11}$$

Next, the angular flux, the boundary source and the response function are expanded in mth order shifted Legendre polynomial series over a predetermined time interval $[0,T]$:

$$\psi(z,\mu,t)=\sum_{\ell=0}^{m}\psi_\ell(z,\mu)P_\ell(t) \tag{12}$$

$$\Gamma_0(t)=\sum_{\ell=0}^{m}\gamma_\ell P_\ell(t) \tag{13}$$

$$R_X(z,\mu,t)=\sum_{\ell=0}^{m}r_{X\ell}(z,\mu)P_\ell(t),\quad X=S,V. \tag{14}$$

The expansion coefficients are given by

$$\psi_\ell(z,\mu)=\frac{T}{2\ell+1}\int_0^T\psi(z,\mu,t')P_\ell(t')dt' \tag{15}$$

$$\gamma_\ell=\frac{T}{2\ell+1}\int_0^T\Gamma_0(t')P_\ell(t')dt' \tag{16}$$

$$r_{X\ell}(z,\mu)=\frac{T}{2\ell+1}\int_0^T R_X(z,\mu,t)P_\ell(t)dt,\quad X=S,V. \tag{17}$$

Inserting these expansions into Eq. (8) yields

$$\sum_{\ell=0}^{m}\psi_\ell(z,\mu)P_\ell(t)=\frac{1}{2}\sum_{\ell=0}^{m}\sum_{\ell=0}^{m}\gamma_\ell r_{S\ell}(z,\mu)\int_0^t d\tau P_\ell(t-\tau)P_\ell(\tau) +\frac{1}{2}Q_0\sum_{\ell=0}^{m}r_{V\ell}(z,\mu)P_\ell(t). \tag{18}$$

The only approximation that has been introduced to this point is the truncation of the Legendre series at the mth order; as m tends to infinity, the solution of Eq. (7) will, in theory, converge to the solution of Eq. (1).

It has been shown (Chang et al., 1987) that polynomial-based convolution integrals such as the one appearing in the first term of the right-hand-side of Eq. (18) can be evaluated explicitly by writing the Legendre series as a power series. This can be accomplished by introducing the basis transformation matrix, $[\mathbf{T}_m]_\Theta^P$, to write the expansions in terms of the standard basis, $\Theta=\{1,t,t^2,\ldots,t^m\}$, rather than the shifted Legendre basis, $P=\{P_0(t),P_1(t),P_2(t),\ldots,P_m(t)\}$. The polynomials coefficients in the Θ basis can be computed by

$$\begin{pmatrix}\bar{a}_0\\ \bar{a}_1\\ \vdots\\ \bar{a}_m\end{pmatrix}=[\mathbf{T}_m]_\Theta^P\begin{pmatrix}a_0\\ a_1\\ \vdots\\ a_m\end{pmatrix} \tag{19}$$

where a_i represents the expansion coefficients γ_i or g_i. Eq. (18) can therefore be expressed as

$$\sum_{\ell=0}^{m}\psi_\ell(z,\mu)P_\ell(t)=\frac{1}{2}\sum_{\ell=0}^{m}\sum_{\ell'=0}^{m}\bar{\gamma}_\ell\bar{r}_{S\ell}(z,\mu)\int_0^t(t-\tau)^\ell\tau^{\ell'}d\tau +\frac{1}{2}Q_0\sum_{\ell=0}^{m}r_{V\ell}(z,\mu)P_\ell(t) \tag{20}$$