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Higher order mode analyses in Feynman-α method

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ABSTRACT

Using a generalized formula for the space and energy dependent Feynman-α method, which was originally derived by Endo et al. and Muñoz-Cobo et al., the effect of higher order modes of the α-mode eigenvalue problem on the Feynman *Y* function has been investigated. To deal with a large number of higher order modes, the diffusion approximation is adopted instead of the transport theory for a one-dimensional homogeneous infinite slab. By making a transport correction to low order mode eigenvalues and eigenfunctions, the formula can accurately reproduce the Monte Carlo simulation results of the Feynman-α method. By virtue of these efforts, an accurate numerical application of the generalized formula, which has not been performed due to the difficulty in solving the higher order α-mode eigenvalue problem, has been made possible. Sample numerical examples for a near-critical system and a deeply-subcritical system quantitatively demonstrate how the Feynman *Y* functions are decomposed into the higher order mode components. While the higher order mode components in the Feynman *Y* function can be negligible in a near-critical system, the Feynman *Y* function in a deeply-subcritical system is found to be severely contaminated by the higher order modes.

1. Introduction

The Feynman-α method is one of representative reactor noise methods for determining the subcriticality of a neutron multiplying system (Feynman et al., 1956). The method deduces an α value (prompt neutron time decay constant) from the variance-to-mean ratio of neutron counts. The basic theory of the method was originally derived for a space and energy independent situation. In a realistic neutron multiplying system, however, there exists the space and energy dependence in neutron flux, and thus the effect of higher order mode neutron flux needs to be considered for accurate estimation of subcriticality. The Feynman-α method is known to give accurate α values in near-critical but subcritical multiplying systems (Misawa et al., 1990; Tonoike et al., 2004). In near-critical systems, the ratios of α values of the higher order modes to that of the fundamental mode become large. In this case, the effect of the higher order modes is insignificant. On the other hand, α values measured by the Feynman-α method differ from the fundamental mode α value as the subcriticality becomes larger (Misawa et al., 1990). This is due to the fact that the ratios of α values of higher order modes to that of the fundamental mode are small and the zeffects of the higher order modes become significant. Therefore, for accurate estimation of subcriticality, it is important to quantify the effects of higher order modes in a subcriticality measurement with the Feynman-α method.

The space and energy dependent theoretical formula for the Feynman-α method was derived by Endo et al. (2006), Ballester and Muñoz-Cobo (2005) and Muñoz-Cobo et al. (2011). Endo et al. derived a Green's function defining a neutron density at a certain point in four-dimensional phase space (space, energy, direction and time) caused by a neutron born at another point. Using the Green's function, a correlated pair-detection probability was derived and eventually a theoretical formula for the space and energy dependent Feynman-α method was derived by double integration of the detection times. With this theoretical formula, it is expected to decompose a Feynman variance-to-mean ratio (or Feynman *Y* function) into the fundamental mode and other higher order mode components. On the other hand, Ballester et al. and Muñoz-Cobo et al. derived a similar but seemingly different formula for the Feynman *Y* function by using stochastic transport theory and then expanding it in alpha modes. However, no comprehensive numerical example for the space and energy dependent Feynman-α method has been presented although some related attempts have been performed thus far (Ballester et al., 2005; Muñoz-Cobo et al., 2011). It may be due to the difficulty that the formula requires the higher order eigenfunctions and eigenvalues of the α-mode neutron transport eigen equation. It is anticipated that a considerable number of higher order modes are required especially for a deeply-subcritical system. Meanwhile, the higher the order, the more difficult it is to obtain the

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eigenfunction. It is important to present numerical examples using the formula for a Feynman Y function, thereby knowing how a Feynman Y function is influenced by higher order modes in a subcriticality measurement.

In this paper, the author attempts to verify whether a Feynman Y function obtained by a Monte Carlo simulation of the Feynman-α method can be reproduced by summing a sufficient number of higher order mode components according to the theoretical formula. Then, how the Feynman Y function is influenced by higher order modes is discussed.

2. Formula for space and energy dependent Feynman-α method

In this section, the formula for the space and energy dependent Feynman-α method derived by Endo et al. (2006) and Muñoz-Cobo et al. (2011) is briefly explained.

Let us consider a subcritical neutron multiplying system having a finite geometry where neutrons are emitted from an external neutron source by following the Poisson process. For simplicity, it is assumed that only one neutron is emitted from the external neutron source at one time. Thus, there is no need for considering the correlation between multiple source neutrons generated at one time. Furthermore, ν (the number of neutrons emitted per fission) is assumed to be a non-stochastic fixed value whereas it is actually a random variable. The probability that one neutron is detected during dt_1 about t_1 is given by

$$P(t_1)dt_1 = C_R dt_1, \tag{1}$$

where

$$C_R = \sum_{n=0}^{\infty} \frac{S_n D_n}{\alpha_n} \ (\text{neutron count rate}), \tag{2}$$

$$S_n = \int_V dV S(\vec{r}) \Psi^*_{s,n}(\vec{r}), \tag{3}$$

$$\Psi^*_{s,n}(\vec{r}) = \int_0^{\infty} dE \int_{4\pi} d\Omega \frac{\chi_s(E)}{4\pi} \psi^*_n(\vec{r}, E, \vec{\Omega}), \tag{4}$$

$$D_n = \int_V dV \int_0^{\infty} dE \int_{4\pi} d\Omega \Sigma_d(\vec{r}, E) \psi_n(\vec{r}, E, \vec{\Omega}), \tag{5}$$

α_n = prompt neutron time decay constant in the nth mode, $S(\vec{r})$ = spatial distribution of the external neutron source intensity, $\chi_s(E)$ = energy spectrum of external neutron source, $\Sigma_d(\vec{r}, E)$ = cross section of detector, $\psi^*_n(\vec{r}, E, \vec{\Omega})$ = nth mode eigenfunction of α-mode adjoint transport equation, $\psi_n(\vec{r}, E, \vec{\Omega})$ = nth mode eigenfunction of α-mode forward transport equation, and the volume integral is performed over the whole volume. The α-mode forward transport equation for the nth mode is defined as

$$\begin{aligned} &\vec{\Omega} \cdot \nabla \psi_n(\vec{r}, E, \vec{\Omega}) + \Sigma_t(\vec{r}, E) \psi_n(\vec{r}, E, \vec{\Omega}) - \int_0^{\infty} dE' \int_{4\pi} d\Omega' \\ &\quad \times \Sigma_s(\vec{r}, E' \to E, \vec{\Omega}' \to \vec{\Omega}) \psi_n(\vec{r}, E', \vec{\Omega}') - \frac{\chi_f(E)}{4\pi} \int_0^{\infty} dE' \\ &\quad \times \int_{4\pi} d\Omega' \nu \Sigma_f(\vec{r}, E') \psi_n(\vec{r}, E', \vec{\Omega}') = \frac{\alpha_n}{\upsilon(E)} \psi_n(\vec{r}, E, \vec{\Omega}), \end{aligned} \tag{6}$$

where $\upsilon(E)$ is the neutron velocity and other notations are standard within the nuclear engineering community. The α-mode adjoint transport equation for the nth mode is defined as

$$\begin{aligned} &-\vec{\Omega} \cdot \nabla \psi^*_n(\vec{r}, E, \vec{\Omega}) + \Sigma_t(\vec{r}, E) \psi^*_n(\vec{r}, E, \vec{\Omega}) - \int_0^{\infty} dE' \int_{4\pi} d\Omega' \\ &\quad \times \Sigma_s(\vec{r}, E \to E', \vec{\Omega} \to \vec{\Omega}') \psi^*_n(\vec{r}, E', \vec{\Omega}') - \frac{\nu \Sigma_f(\vec{r}, E)}{4\pi} \int_0^{\infty} dE' \\ &\quad \times \int_{4\pi} d\Omega' \chi_f(E') \psi^*_n(\vec{r}, E', \vec{\Omega}') = \frac{\alpha_n}{\upsilon(E)} \psi^*_n(\vec{r}, E, \vec{\Omega}). \end{aligned} \tag{7}$$

The α-mode eigenfunctions have an orthogonality condition as follows:

$$\int_V dV \int_0^{\infty} dE \int_{4\pi} d\Omega \frac{1}{\upsilon(E)} \psi^*_m(\vec{r}, E, \vec{\Omega}) \psi_n(\vec{r}, E, \vec{\Omega}) = \delta_{mn}, \tag{8}$$

where δ_{mn} is the Kronecker delta. The orthogonality condition (i.e., Eq. (8)) and D_n in Eq. (5) are slightly changed from those in the paper by Endo et al. (2006) in dealing with the velocity.

The probability of a pair of neutron counts during the time intervals dt_1 about t_1 and dt_2 about t_2 ($t_2 > t_1$) resulting from a fission event is given by (Endo et al., 2006)

$$P_{2,c}(t_1, t_2) dt_1 \, dt_2 = \sum_{\ell=0}^{\infty} \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} \frac{S_\ell F_{\ell \to mn} D_m D_n}{\alpha_\ell (\alpha_m + \alpha_n)} e^{-\alpha_n (t_2 - t_1)} dt_1 \, dt_2, \tag{9}$$

where

$$\begin{aligned} F_{\ell \to mn} = &\int_V dV \int_0^{\infty} dE \int_{4\pi} d\Omega \nu(\nu - 1) \\ &\times \Sigma_f(\vec{r}, E) \psi_\ell(\vec{r}, E, \vec{\Omega}) \Psi^*_{f,m}(\vec{r}) \Psi^*_{f,n}(\vec{r}), \end{aligned} \tag{10}$$

$$\Psi^*_{f,n}(\vec{r}) = \int_0^{\infty} dE \int_{4\pi} d\Omega \frac{\chi_f(E)}{4\pi} \psi^*_n(\vec{r}, E, \vec{\Omega}). \tag{11}$$

Eq. (9) for the probability of correlated counts from fissions events is the same in principle as Eq. (67) in the paper of Muñoz-Cobo et al. (2011). The probability of a uncorrelated pair detection resulting from two independent neutron families is also given by

$$P_{2,u}(t_1, t_2) dt_1 \, dt_2 = P_1(t_1) P_1(t_2) dt_1 \, dt_2. \tag{12}$$

The expected number of pairs detected during the counting gate width T is given by

$$\left\langle \frac{C(T)(C(T) - 1)}{2} \right\rangle = \int_0^T dt_2 \int_0^{t_2} (P_{2,u}(t_1, t_2) + P_{2,c}(t_1, t_2)) dt_1, \tag{13}$$

where the bracket $\langle\rangle$ stands for an average over time. As a result, $Y(T)$ in the Feynman-α method taking into account the space and energy dependence is given by (Endo et al., 2006)

$$\begin{aligned} Y(T) &= \frac{\langle C(T)(C(T) - 1) \rangle - \langle C(T) \rangle^2}{\langle C(T) \rangle} = \frac{\langle C(T)^2 \rangle - \langle C(T) \rangle^2}{\langle C(T) \rangle} - 1 \\ &= \sum_{n=0}^{\infty} Y_n(T), \end{aligned} \tag{14}$$

where

$$Y_n(T) = \frac{2}{C_R} \sum_{m=0}^{\infty} \left(\sum_{\ell=0}^{\infty} \frac{S_\ell F_{\ell \to mn}}{\alpha_\ell} \right) \frac{D_m D_n}{\alpha_n (\alpha_m + \alpha_n)} \left(1 - \frac{1 - e^{-\alpha_n T}}{\alpha_n T} \right). \tag{15}$$

However, Muñoz-Cobo et al. (2011) derived a similar but different formula for the Feynman Y function as

$$Y(T) = \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} Y_{m,n}(T), \tag{16}$$

where

$$Y_{m,n}(T) = \frac{D_m D_n}{C_R \alpha_m \alpha_n} \left(\sum_{\ell=0}^{\infty} \frac{S_\ell F_{\ell \to mn}}{\alpha_\ell} \right) \left(1 - \frac{\alpha_m^2 (1 - e^{-\alpha_n T}) + \alpha_n^2 (1 - e^{-\alpha_m T})}{\alpha_m \alpha_n (\alpha_m + \alpha_n) T} \right). \tag{17}$$