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# Multigroup pin power reconstruction with two-dimensional source expansion and corner flux discontinuity

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ABSTRACT

A pin power reconstruction method that is readily applicable to multigroup problems with superior accuracy is presented for applications involving rectangular fuel assemblies. It employs a two-dimensional (2D), fourth order Legendre expansion of the source distribution that naturally leads to a group-decoupled, 2D semi-analytic solution of the neutron diffusion equation. The four surface average currents and four corner fluxes are used as the boundary conditions to uniquely specify the homogenous solution. The corner fluxes and source expansion coefficients are iteratively determined using the condition of corner point balance and the orthogonal property of the Lengedre functions. Corner discontinuity is incorporated in the calculation of the corner fluxes which turns out to be very effective in the cases of enrichment zoning. The accuracy of the proposed method is assessed by performing the two-step core calculations for the L336C5, C5G7MOX, and MOX core transient benchmark problems and then by comparing with the direct whole-core transport solutions. The results indicate that the proposed method is as accurate as the fully analytic method and works well irrespective the number of groups. However, it is also noted that somewhat larger errors are inevitable at the peripheral assemblies near the reflector in which the error associated with *a prioi* generation of the homogenized cross-sections and form functions is not trivial.



## 1. Introduction

With the increasing need for multigroup reactor calculation, there has been renewed interest in the multigroup nodal codes (Bahadir et al., 2005) and methods (Aragones et al., 2007; Yoon and Joo, 2008) in recent years. Multigroup capability is needed for complex core designs or highly leaking cores in which *a priori* few-group condensation can suffer large errors due to the considerably different spectrum actually realized in the core. Examples are heavily mixed oxide (MOX) loaded cores and gas cooled reactor cores as well as fast reactor cores.

In the multigroup core calculation, the transverse-integrated nodal methods (TINMs) are normally chosen because of their superior calculation efficiency and accuracy. Note that although the multi-dimensional nodal methods such as the analytic function expansion method (Noh and Cho, 1994), which do not involve transverse-integration, are in principle more accurate than TINMs, there is no strong need for such methods because the difference in accuracy between the two classes of nodal methods is negligible as long as four nodes per assembly are used in the nodal calculation as required to capture properly the intra-nodal burnup distribution. Once a TINM is employed in the nodal core calculation, a two-dimensional flux calculation need to follow the nodal calculation in order to reconstruct the pin power distribution. Although the reconstructed pin power distribution obtainable in the standard two-step core calculation procedure may not be as accurate as the direct pin-by-pin transport solution, sufficiently accurate pin power distributions can be obtained as long as a sophisticated reconstruction method and a nodal method are implemented. This paper deals with a new accurate method for pin power reconstruction which can be applied readily to multigroup problems.

There are numerous pin power reconstruction methods developed for few-group problems. These methods differ mostly in how to represent and generate the homogeneous flux. In earlier days, two-dimensional polynomials (Koebke and Wagner, 1977) were used in the representation of the intranodal flux distribution and later exponential functions were introduced to augment the thermal flux variation (Koebke and Hetzelt, 1985; Rempe et al., 1988). More recently, analytic functions were used in both energy groups (Böer and Finnemann, 1992). The analytic method employs the analytic solution of the two-dimensional (2D) Helmholtz equation satisfying a given set of boundary conditions in an isolated two-dimensional geometry. The analytic solution renders superior accuracy compared to polynomial based methods. As the boundary conditions, Böer and Finnemann used four surface average fluxes and four corner fluxes per energy group (Böer and Finnemann, 1992).

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The use of surface fluxes, however, does not guarantee that the resulting two-dimensional homogenous solution satisfy the nodal balance condition given by the surface currents and the node-average flux determined in the nodal calculation. Therefore, the 2D solution is not fully consistent with the nodal solution. We had introduced another analytic method which uses the four surface currents instead of the surface fluxes to produce a consistent 2D solution (Joo et al., 1999). Note that once the four surface currents used as the boundary condition for the solution of the 2D neutron diffusion equation, the average of the resulting 2D flux distribution is guaranteed to be the same as the node average flux of the nodal solution. The analytic 2D solution method is, however, not readily applicable to multigroup(MG) problems because of the diagonalization problem of the $G \times G$ 2D neutron diffusion equations.

Bahadir and Lindahl (2006) recently introduced a multigroup pin power calculation method which involves submesh solutions of the SIMULATE-4 code. In this method, an assembly mesh is divided into $N \times N$ submeshes and four exponential functions, $e^{\kappa_x x}, e^{-\kappa_x x}, e^{\kappa_y y}$ and $e^{-\kappa_y y}$, augmented by quadratic polynomials are used to approximate the 2D intranodal flux shape. The flux representation, however, lacks the cross-terms and thus cannot be a good approximation for large node sizes. Nonetheless, since the primary purpose of the submesh method of the SIMULATE-4 is rehomogenization which requires smaller submeshes (typically $N = 5$), the low order approximation may not be a significant problem. However, for the traditional nodal method which does not use the submesh scheme, a better 2D solution method that is readily applicable to MG problems is needed. In order to meet this need, a 2D semi-analytic solution is derived in this paper by introducing a 2D Legendre function expansion of all the source terms.

In the following, the basic formulation of the 2D semi-analytic solution is derived first which employs four surface average currents and four corner fluxes as the boundary condition in order to retain consistency with the nodal solution. Since the corner fluxes need to be newly determined in the 2D flux calculation, a consistent method for determining the corner fluxes by accounting for corner discontinuity factors (CDFs) is developed as well. The source expansion method basically requires an iterative scheme to approximate the source distribution from the semi-analytic solution. In Section 3, the iterative calculation scheme is described together with the whole calculation sequence of generating the heterogeneous pin power distribution. In Section 4, the performance of the proposed method is first examined for a typical two-group (2G), 2D pin power benchmark problem L336C5 (Cavarec et al., 1994) and then for two MG problems: the C5G7 benchmark (Lewis et al., 2001) and a realistic MG MOX core problem given in the MOX transient benchmark (Kozlowski and Downar, 2006). Comparisons with the heterogeneous solution or the direct whole-core transport solutions will be made to assess the pin error errors.

## 2. Semi-analytic solution with 2D source expansion

Given the finite number of boundary conditions such as four surface average currents and four corner fluxes, a complete analytic solution to the 2D MG neutron diffusions can be obtained in principle irrespective of the number of groups through the diagonalization process which removes the group coupling between the groupwise neutron diffusion equations. The analytic solution for each modal flux then can be obtained with the same number of terms as the boundary conditions. In our choice of boundary conditions, the analytic solution for each modal flux consists of eight terms. However, since the diagonalization process is cumbersome and the analytic solution would be difficult to apply to find groupwise corner fluxes, we here seek semi-analytic solution by decoupling the group dependence.

By the semi-analytic solution, it is meant that the analytic solution is obtained for the 2D neutron diffusion equation after moving all the source terms to the right hand side (RHS) and then approximating the source distribution by a polynomial. By the group decoupling which is achieved by treating the source distribution as known from the previous iterative solution, the analytic solution can be obtained easily for each group as a combination of an exponential homogeneous solution and a polynomial particular solution. Because of the polynomial expansion with a finite number of terms, however, the semi-analytic solution would always be less accurate than the fully analytic solution. The accuracy of the semi-analytic solution would increase as the number of terms in the source polynomial increases.

The homogeneous solution of the 2D neutron diffusion equation is the solution of the Helmholtz equation which consists of an infinite number of $\cosh \kappa(x\cos\theta + y\sin\theta)$ and $\sinh \kappa(x\cos\theta + y\sin\theta)$ function pairs with an arbitrary angle $\theta$. With a finite number of boundary conditions, however, specific values of $\alpha$ need to be chosen. Böer and Finnemann (1992) chose 0°, 45°, 90° and 135° for $\theta$ to associate the coefficients of the homogeneous solution coefficients with the surface average and corner fluxes that constitute a total of 8 boundary conditions. The same homogenous solution form will be used in the following derivation of the 2D semi-analytic solution.

### 2.1. Two-dimensional semi-analytic solution

After integrating the 3D neutron diffusion equation axially over a plane of thickness, $h_z$, the following 2D balance equation is obtained for each group with all the source terms moved to the right hand side (RHS):

$$D_g \nabla^2 \phi_g(x,y) + \Sigma_{rg}\phi_g(x,y) = \lambda\chi_g \sum_{g'} \nu\Sigma_{fg'}\phi_{g'}(x,y) + \sum_{g' \neq g} \Sigma_{g'g}\phi_{g'}(x,y) - L_{gz}(x,y) \quad (1)$$

where the axial transverse leakage is defined as $L_{gz}(x,y) = \frac{1}{h_z}(J^T_{gz}(x,y) - J^B_{gz}(x,y))$ in term of the currents at the top and bottom surfaces of the plane and $\lambda$ is the inverse of the multiplication factor. The entire RHS term can be approximated by a polynomial of two spatial coordinate variables. The polynomial for a 2D shape might be iteratively updated. With such a polynomial approximation, the solution of Eq. (1) would be obtained straightforwardly for a square node whose width is $h$.

Since it is advantageous to use the Legendre polynomial for the polynomial approximation owing to its orthogonal property, we first normalize the independent variable such that it varies from −1.0 to 1.0 in the node. This leads to the following equation with the group index $g$ omitted:

$$-\frac{4D}{h^2}\left(\frac{\partial^2}{\partial\xi^2} + \frac{\partial^2}{\partial\eta^2}\right)\phi(\xi,\eta) + \Sigma_r\phi(\xi,\eta) = Q(\xi,\eta) \quad (2)$$

where $\xi = \frac{2x}{h}, \eta = \frac{2y}{h}$, and $Q(\xi,\eta)$ represents the distribution of the entire sources. As the polynomial approximation to the source distribution, we use a quartic polynomial given in terms of Legendre polynomials:

$$Q(\xi,\eta) = \sum_{\substack{i=0 \\ i+j \le 4}}^{4} \sum_{\substack{j=0 \\ \le 4}}^{4} q_{i,j} P_i(\xi) P_j(\eta) \quad (3)$$

where $P_i(\xi)$ is the $i$th order Legendre polynomial. Note that this 15-term polynomial contains fourth order cross-terms such as $P_1(\xi)P_3(\eta)$ and $P_2(\xi)P_2(\eta)$ and represent a quartic variation in each direction. In the following, it will be assumed that the source term