Computers and Chemical Engineering 35 (2011) 1200–1214

Contents lists available at ScienceDirect

Computers and Chemical Engineering

journal homepage: www.elsevier.com/locate/compchemeng

# Direct simulations of spherical particle motion in Bingham liquids

Prashant, J.J. Derksen*

*Chemical & Materials Engineering Department, University of Alberta, ECERF 7-028, Edmonton, Alberta T6G 2G6, Canada*

ARTICLE INFO

*Article history:*
Received 21 October 2009
Received in revised form 3 September 2010
Accepted 6 September 2010
Available online 15 September 2010

*Keywords:*
Bingham liquid
Simulation
Lattice-Boltzmann
Lid-driven cavity
Sedimentation

ABSTRACT

The present work deals with the development of a direct simulation strategy for solving the motion of spherical particles in a Bingham liquid. The simulating strategy is based on a lattice-Boltzmann flow solver and the dual-viscosity Bingham model. Validation of the strategy is first performed for single phase (lid-driven cavity flow) and then for two phase flows. Lid-driven cavity flow results illustrate the flow's response to an increase of the yield stress. We show how the settling velocity of a single sphere sedimenting in a Bingham liquid is influenced by the yield stress of the liquid. The hydrodynamic interactions between two spheres are studied at low and moderate Reynolds number. At low Reynolds number, two spheres settle with equal velocity. At moderate Reynolds number, the yield effects are softened and the trailing sphere approaches the leading sphere until collision occurs.



## 1. Introduction

Bingham liquids, a special subclass of viscoplastic liquids, possess a yield stress which must be exceeded before the liquid shows any significant deformation. By virtue of its yield stress, a Bingham liquid is capable of trapping an embedded particle for a long time. For example, drilling liquids used in petroleum industry posses a yield stress and prevent the settling of rock debris when their circulation is stopped (Peysson, 2004). The presence of a yield stress in various industrial liquids is critical to solid–liquid suspensions. In oil sand operations (Masliyah, Zhou, Xu, Czarnecki, & Hamza, 2004), clay particles get surface activated in the presence of water and make a complex clay–water suspension. This complex suspension possesses a yield stress which is relevant for the design, operation and efficiency of oil sands processing, especially in those parts of the process related to separation and to tailings. If the net gravity force acting on inert particles (sand, bitumen drops) is not enough to overcome the yield stress, they are trapped in the clay network hindering gravity based separation.

Research studies focused on spheres sedimenting in Bingham liquids date back several decades. Ansley and Smith (1967) postulated the shape and extent of yielded/unyielded regions surrounding the sphere using slip line theory. In a classical work, Beris, Tsamopoulos, Armstrong, and Brown (1985) numerically determined the velocity field, pressure field, shape of the yield surfaces and drag coefficient for the creeping flow around a sphere in an unbounded Bingham liquid. Blackery and Mitsoulis (1997) reported drag coefficients for bounded flows with various tube/sphere diameter ratios. More recently Liu, Muller, and Denn (2002) and Yu and Wachs (2007) obtained the shape and extent of yielded/unyielded regions for such flow systems. Turning the attention towards sedimentation of more than one sphere, one finds few results for Bingham liquids, probably due to the complexity associated with two sphere motion in addition to the discontinuous nature of Bingham liquid model. Liu, Muller, and Denn (2003) numerically investigated the creeping flow of two identical spheres falling collinearly along the axis of a circular cylinder in a Bingham liquid. They calculated the yield surfaces as a function of the ratio of the center to center distance over the radius of the spheres and further predicted a plug like (unyielded) region between the two spheres along the symmetry axis. In an experimental work, Merkak, Jossic, and Magnin (2006) reported an appreciable hydrodynamic interaction between two spheres falling one above the other. They proposed drag coefficient correlations and showed that the yield effect of viscoplastic liquids reduces the degree of interaction compared to sedimentation in Newtonian liquid. Yu and Wachs (2007) examined the motion of two spheres translating along the axis of a tube at low Reynolds number and predicted a higher velocity of two spheres than a single sphere due to the hydrodynamic interaction.

One of the difficulties encountered in implementing the Bingham model in a computer code is its non-differentiable form. There are mainly three approaches which have been used in the literature to counter these problems: the dual-viscosity model (Beverly & Tanner, 1992; O'Donovan & Tanner, 1984), regularization methods (Mitsoulis & Zisis, 2001; Papanastasiou, 1987), and variational inequality based methods (Vola, Boscardin, & Latche, 2003; Yu & Wachs, 2007). The first two methods approximate the Bingham model by considering the solid region as a highly viscous mate-

* Corresponding author. Tel.: +1 780 492 5267; fax: +1 780 492 2881.
*E-mail address:* jos@ualberta.ca (J.J. Derksen).

0098-1354/$ – see front matter © 2010 Elsevier Ltd. All rights reserved.
doi:10.1016/j.compchemeng.2010.09.002

rial. The variational inequality method is rigorously equivalent to the original Bingham model, and is implemented by introducing Lagrange multipliers.

The present work concentrates on the sedimentation of particles in Bingham liquid using the lattice-Boltzmann method (LBM) as flow solver. First we discuss and analyse the results of single sphere sedimentation at low Reynolds number. We then investigate the hydrodynamic interaction between the two spheres falling along the axis of symmetry at low and moderate Reynolds number. Hydrodynamic interaction is interpreted in terms of settling velocity, flow field and attraction between the spheres. The (benchmark) results presented in this paper are meant to assess the accuracy and potential of the computational method so that (in future work) the method can be used for flow systems relevant to industrial and environmental processes, such as the simulation of dense solid–liquid suspensions involving Bingham liquids.

This paper is organized as follows: in Section 2 a brief introduction to LBM, the Bingham model and dimensionless numbers is provided. Validation of the numerical procedure is accomplished by comparing results for lid-driven cavity flow with results from the literature (Section 3). In Section 4 we study the single sphere sedimentation in Bingham liquid in a confined domain and report the effects of yield stress on the settling velocity. In Section 5, we examine the sedimentation of two spheres (one above the other) in Bingham liquid at low and moderate Reynolds number. The concluding remarks are provided in Section 6.

## 2. Numerical model

### 2.1. Flow solver

The lattice-Boltzmann method is a well established and frequently used method for simulating liquid flows. In principle it has a second order accuracy in space and time and is particularly regarded an efficient flow solver for flows involving interfacial dynamics and complex geometries (Chen & Doolen, 1998). LBM originated from lattice gas automata in which liquid particles are distributed on a lattice of nodes. Each liquid particle has certain directions of velocities at each node. At each time step a liquid particle is involved in two sequential processes: streaming and collision. In the streaming process, the liquid particle moves from one node to the nearest node in the direction of its velocity and in collision it interacts with other liquid particles reaching the same node and changes its velocity as per collision rules. In this work, we make use of the formulation by Eggels and Somers (1995) which is a D3Q18 model (three-dimensional, 18 velocities). In LBM the units of distance and time are the lattice spacing and the time step, respectively. All the liquid properties and flow variables are scaled to dimensionless quantities within certain ranges (e.g. for density and kinematic viscosity: $\rho \sim 8$, $0.25 > \nu > 10^{-5}$) (Eggels & Somers, 1995). The bounce-back scheme is a popular way to mimic no-slip boundary conditions at plane walls. In this scheme, the liquid particle is reflected back to the node it comes from. Explicitly applying zero velocity on boundaries is an alternative to retrieve the no-slip condition. No-slip boundary condition at a curved boundary is achieved by a forcing method (Derksen & Van den Akker, 1999), also known as immersed boundary method.

### 2.2. Bingham model

Viscoplastic liquids possess a yield stress ($\tau_0$) which must be exceeded before the fluid shows any significant deformation. The Bingham model, one of the simplest rheological models, is used to describe the flow properties of liquid with a yield stress $\tau_0$. The deformation rate remains zero and the material behaves like a solid

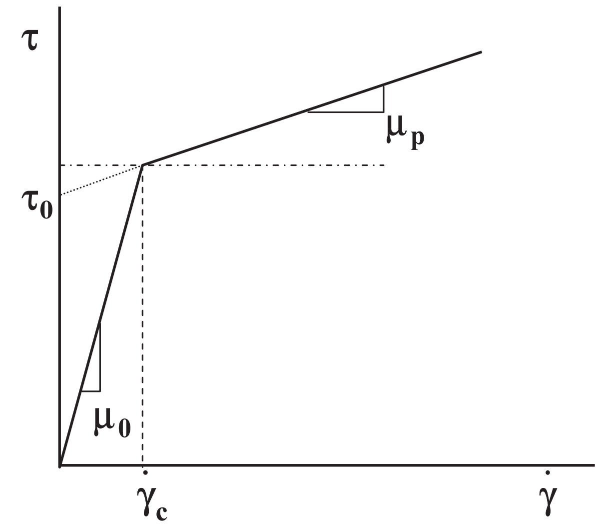


**Fig. 1.** One-dimensional representation of dual-viscosity Bingham model.

until the stress exceeds the yield limit of the liquid. In a generalized manner, the constitutive equations for a Bingham liquid can be written as

$$\tau_{ij} = 2\left(\frac{\tau_0}{\dot{\gamma}} + \mu_p\right) d_{ij} \quad |\tau| > \tau_0 \tag{1}$$

$$d_{ij} = 0 \quad |\tau| < \tau_0 \tag{2}$$

In the above expressions, $\tau_{ij}$ is deviatoric part of whole stress tensor $\sigma_{ij}$, $\dot{\gamma}$ is the deformation rate ($\dot{\gamma} = \sqrt{d_{ij}d_{ij}}$), $d_{ij}$ is the rate of deformation tensor $[1/2((\partial u_i/\partial x_j)+(\partial u_j/\partial x_i))]$, $\mu_p$ is the plastic viscosity, and $|\tau|$ is the magnitude of the shear stress ($|\tau| = \sqrt{\tau_{ij}\tau_{ij}}$).

In the present work, a dual-viscosity model is used to mimic Bingham liquids (Beverly & Tanner, 1992) because of its less complex structure and easy implementation within the lattice-Boltzmann scheme, which essentially is a viscous flow solver. In this model the region around zero shear rate is characterized by a highly viscous material with viscosity $\mu_0$. At higher shear the actual Bingham rheology is represented by a much lower plastic viscosity $\mu_p$. The one-dimensional dual-viscosity Bingham rheology is shown in Fig. 1. The transition (from high to low viscosity) gives rise to a critical shear rate $\dot{\gamma}_c = \tau_0/(\mu_0 - \mu_p)$. When the shear rate ($\dot{\gamma}$) becomes greater than critical shear rate, material is considered yielded. Thus the criterion of yielded and unyielded regions is defined as

$$\dot{\gamma} > \dot{\gamma}_c \rightarrow \text{yielded} \tag{3}$$

$$\dot{\gamma} \leq \dot{\gamma}_c \rightarrow \text{unyielded} \tag{4}$$

In the dual-viscosity model, the apparent viscosity ($\mu_a$) and shear stress ($\tau_{ij}$) of the material are written as

$$\mu_a = \mu_0 \quad \dot{\gamma} \leq \dot{\gamma}_c \tag{5}$$

$$\mu_a = \mu_p + \frac{\tau_0}{\dot{\gamma}} \quad \dot{\gamma} > \dot{\gamma}_c \tag{6}$$

$$\tau_{ij} = 2\mu_a d_{ij} \tag{7}$$

### 2.3. Dimensionless numbers

As the two dimensionless numbers that govern the Bingham liquid flow system we chose a Reynolds number and a Bingham number:

$$\text{Reynolds number } Re = \frac{\rho_f U_c L_c}{\mu_p} \tag{8}$$