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Performance analysis of sequential Carnot cycles with finite heat sources and heat sinks and its application in organic Rankine cycles

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A B S T R A C T

The sequential Carnot cycle, where a number of single Carnot cycles are connected in parallel, has been researched for renewable thermal energy utilization because the characteristics of its low grade heat source, are different from those of a conventional one. In this paper, the thermodynamic analysis is conducted on the sequential Carnot cycle which uses not only finite heat sources but also finite heat sinks. Equations for efficiency and power from the whole sequential system are derived from the general theory of thermodynamics and heat transfer. Based on the equation from theory, the performance of the sequential system is calculated in various system conditions, including the ideal situation where the system has an infinite number of Carnot cycles and infinite heat exchanger inventory. In addition, the sequential concept is applied to organic Rankine cycles, which is one of the most used thermodynamic systems that generate work from low grade heat sources. From a simple simulation with properties of real working fluids, the performance of a sequential ORC (organic Rankine cycle) is investigated and its result is compared with that of theoretical sequential Carnot cycles.

1. Introduction

Since fossil fuels, which meet most of energy requirements in the world, are not so environmental friendly and their present stocks are finite, various renewable energy sources have been introduced as a promising option to fill increasing demand of worldwide energy [1]. Although some typical renewable energy sources, such as solar photovoltaic energy, wind energy, and hydrogen energy of fuel cells, are directly converted into electricity, a lot of renewable sources, such as solar thermal energy, industrial waste heat, geothermal energy, and ocean thermal energy, still exist in the form of heat [2]. These thermal energy sources, which are categorized as low grade heat sources because they have low temperature and relatively small heat capacity [3], require some thermodynamic cycles to generate a useful power.

Generally, the Carnot cycle has been used for an evaluation of thermal systems as a standard. However, considering the characteristics of renewable heat sources are different from those of conventional sources, it may not be the best choice to use just a single Carnot cycle for analyzing renewable thermal energy systems as it deals with heat sources of infinite heat capacity. Therefore, the sequential Carnot cycle has been suggested, where a number of individual Carnot cycles are arranged in parallel [4]. In contrast to the original Carnot cycle, the sequential cycle considers the temperature change of finite heat sources which occurs at the heat transfer process between the cycle and heat sources. Moreover, in case of a sequential Carnot cycle, the heat transfer rate between the cycle and heat sources is calculated by taking account of temperature difference between them. More detailed information about the system model is explained later.

Although a lot of researchers [5–9] have studied a modified single Carnot cycle to apply it to the actual cycle more practically since Curzon and Ahlborn's research [10], a little research about the sequential Carnot system has been carried out since it was firstly suggested by Ondrechen et al. [4]. Ibrahim and Klein [11] adopted sequential Carnot cycles simply as a reference one to evaluate the performance of some absorption power cycles. Sieniutycz and Spakovsky [12] investigated the maximum power output of a multistage continuous endoreversible Carnot system. They carried out the optimization study by introducing Hamiltonian function. Chen et al. [13] further studied the maximum power of a multistage Carnot system, in which the heat transfer obeys the generalized heat transfer principles. They obtained the numerical solutions by

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Nomenclature

B	availability (W)
C	heat capacity of heat sources (W/K)
N	number of individual cycles in the system
NTU	number of transfer unit (=UA/C)
Q	heat transfer rate (W)
T	temperature (K)
T_R	ratio of outlet temperature to inlet temperature
T_{amb}	ambient temperature (K) (=T_0 = 298.15 K)
UA	product of overall heat transfer coefficient and heat transfer area (W/K)
W	work (W)

Greek symbols

η	efficiency

Subscripts

con	condenser
eva	evaporator
F	final state of heat sources
H	high temperature side, a heat source
i	element of sequence, inlet
ideal,1	first ideal case (infinite N and NTU)
ideal,2	second ideal case (infinite N and finite NTU)
ideal,3	third ideal case (finite N and infinite NTU)
isen	isentropic
I	inlet condition of heat sources
L	low temperature side, a heat sink
o	outlet condition of heat sources
tot	total

using Hamilton-Jacobi-Bellman theory and the dynamic programming algorithm. Baik et al. [14] calculated theoretical maximum power of a heat engine using a low grade heat source of about 100 °C by the sequential Carnot cycle model. They adopted the pattern search algorithm to find out optimized conditions. Although they succeeded in obtaining solutions of their problems in these studies, their process is so complicated that it is hard to understand and be applied to other examples. Ohman and Lundqvist [15,16] also used similar concept to facilitate characterization and comparison of power cycles using low temperature heat sources. They adopted local Carnot efficiency, which corresponds to efficiency of each Carnot cycle in sequential systems, and analyzed several power cycles with published field data. Still, additional mathematical solver, such as EES (engineering equation solver) is needed to obtain the efficiency they suggested as a reference and they considered only an ideal case where more than 100 steps of local efficiency are used. Park and Kim [17] analyzed the thermodynamic performance of sequential Carnot cycles and conducted optimization process to maximize generated power from the system. They established useful equations so that the performance of the sequential system is obtained and optimized for certain conditions analytically. Nevertheless, their work about sequential Carnot cycles deal with the case of finite heat sources and infinite heat sinks. The case of both finite heat sources and finite heat sinks has hardly been covered even though it is more practical one.

In this paper, the thermodynamic analysis of the sequential Carnot cycle using both a heat source and a heat sink with finite heat capacity is carried out. The performance of system is derived as equations based on fundamental theory of thermodynamics and heat transfer. As this system was suggested as an alternative standard cycle, some useful expressions about the performance in an ideal case, are provided. Also, to narrow the gap between the theoretical research and practical situations, the sequential concept is applied to an ORC (organic Rankine cycle) which is well known for the target system of low grade heat sources. From a simple simulation, the performance of a sequential ORC is obtained and it is compared with the result of theoretical analysis.

2. System description and analytical modeling

The schematic diagram of a sequential Carnot cycle model used in this study is illustrated in Fig. 1. As mentioned earlier, this cycle consists of N Carnot cycles which are connected in parallel. The heat source, whose temperature is $T_{I,H}$, enters the system and leaves it at the temperature of $T_{F,H}$ after transferring heat to the system. The heat sink, which has finite heat capacity and initial temperature of $T_{I,L}$, absorbs heat from each individual Carnot cycle and its temperature is increased at certain point. Also, UA, which is the product of an overall heat transfer coefficient and heat transfer area, is considered to express the heat exchanger inventory and calculate heat transfer rate between Carnot cycles and heat sources. The friction and loss of pressure and heat which can happen in the system are presumed to be negligible.

Since this system is imaginary and only possible in theory, its design and operation are decided with six variables only, which are N, NTU_H, NTU_L, $T_{I,H}$, $T_{I,L}$, and $T_{F,H}$. But the distribution of NTU among each individual Carnot cycle in sequential systems and the temperature profile of a heat source when its initial and final temperature are fixed can slightly affect the system performance. Therefore, in this study, these two suppositions, which are verified to be optimal from Lagrange multiplier method [18], are used:

1. The heat exchanger inventory (NTU) is uniformly distributed in each Carnot cycles for a sequential system

$$NTU_{H,1} = NTU_{H,2} = \cdots = NTU_{H,N} = \frac{1}{N} NTU_H \tag{1}$$

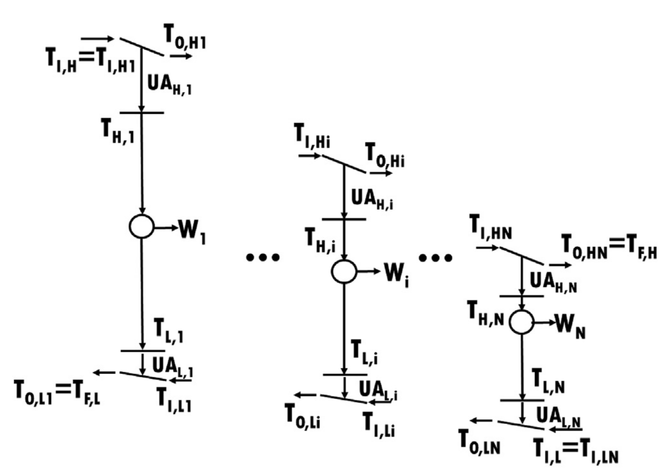

Fig. 1. The schematic diagram of the sequential Carnot cycle model.