Energy 99 (2016) 237–249

Contents lists available at ScienceDirect

Energy

journal homepage: www.elsevier.com/locate/energy

Numerical investigation of magnetic field effects on entropy generation in viscous flow over a stretching cylinder embedded in a porous medium

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ARTICLE INFO

Article history:
Received 4 May 2015
Received in revised form
18 January 2016
Accepted 24 January 2016
Available online 15 February 2016

Keywords:
Stretching cylinder
Magnetic field
Porous medium
Entropy generation

ABSTRACT

The impact of presence of magnetic field on entropy generation in flow and heat transfer of viscous fluid over a horizontal stretching cylinder embedded in a porous medium has been theoretically studied. The problem is modeled mathematically and the governing partial differential equations are converted into non-linear differential equations by using suitable similarity transformations. Numerical solution of the transformed equations is obtained using the matlab built-in routine **bvp4c**. The velocity profiles, temperature distribution, local entropy generation number and Bejan number are plotted for various values of magnetic field parameter, curvature parameter, permeability parameter, Prandtl number, Eckert number, temperature exponent and group parameter. Moreover, the effects of these pertinent parameters on skin friction coefficient and local Nusselt number are also presented through tables. A brief discussion has been given about the impact of these physical parameters on thermo-physical properties.

1. Introduction

The boundary layer flows induced by stretching surfaces have many applications in extrusion processes and are also of theoretical interest. Applications of such flows occur in cooling of metallic plates, drawing of plastic sheets, paper productions, aerodynamics etc. The heat transfer phenomenon is additionally critical in such processes as the nature of last product relies upon the rate of high temperature exchange. Crane [1] carried out the initial study on boundary layer flow caused by the linear stretching of flat surface and obtained the exact analytical solution of the problem. Taking into account his work, several investigators such as Gupta and Gupta [2], Grubka and Bobba [3], Ali [4], Chen [5], Datta et al. [6], Chen and Char [7], Elasbheshy [8], Cortell [9], Liao [10], Ahmer and Ali [11], Ishak et al. [12] extended the work under diverse physical circumstances.

All the above mentioned studies are related to flow and heat transfer characteristic over a flat stretching surface. However, analysis of flow and heat transfer phenomena over stretching cylinder is also important in processes like fiber and wire drawing, hot rolling etc. Based on these applications, Crane [13] initiated the study on boundary layer flow induced due to a stretching cylinder. Crane's work was extended by Wang [14] to examine the flow and heat transfer outside a hollow stretching cylinder. Burde [15] obtained the exact similarity solutions of axisymmetric flow of viscous fluid due to linearly stretching of infinite circular cylinder. Ishak and Nazar [16] numerically investigated the heat transfer characteristic in viscous fluid flow over a stretching cylinder. The effects on magnetic field on flow and heat transfer due to linearly stretching cylinder were discussed by Ishak et al. [17]. Mastroberardino and Paullet [18] proved the existence and uniqueness of the solution for the problem of axisymmetric flow over a permeable stretching cylinder. Weidman and Ali [19] analyzed the aligned and non-aligned stagnation point flow over a stretching cylinder. Wang and Ng [20] extended the Wang's problem [14] by considering the slip effects and found that the presence of slip at the surface of stretching cylinder results in reduction of the magnitudes of velocities. Munawar et al. [21] discussed the flow of a viscous fluid over an oscillatory stretching cylinder by using an implicit finite difference scheme. Vajravelu et al. [22] investigated the axisymmetric hydromagnetic fluid flow and heat transfer over a stretching cylinder by considering prescribed surface

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http://dx.doi.org/10.1016/j.energy.2016.01.067
0360-5442/

temperature and flux. Munawar et al. [23] studied the unsteady boundary-layer flow and heat transfer of a viscous fluid over a stretching cylinder by using both analytical and numerical techniques. The mixed convective flow along a stretching cylinder embedded in a thermally stratified medium was examined by Mukhopadhyay and Ishak [24]. Shateyi and Marewo [25] made use of SRM (successive relaxation method) method to discuss the axisymmetric flow over a stretching cylinder embedded in a porous medium. Si et al. [26] presented the numerical solution of unsteady flow and heat transfer of viscous fluid due to porous cylinder which is stretching and expanding at the same time. Recently, Vajravelu et al. [27] analyzed the flow and heat transfer phenomenon over a permeable stretching cylinder by assuming temperature dependent thermophysical properties.

Entropy generation effects due to flow and heat transfer over flat plates or stretching surfaces have been investigated by many researchers [28–38]. However, a few investigations are found in literature relating to the study of entropy generation effects in flow over stretching cylinders [39,40]. The present article theoretically investigates the effects of magnetic field on entropy production due to viscous fluid flow and heat transfer over a stretching cylinder embedded in a porous medium. The equations are solved numerically and the results are interpreted using graphs and tables.

2. Mathematical formulation

Consider a steady, axisymmetric laminar flow of an incompressible viscous fluid due to continuous stretching of a horizontal cylinder having radius a embedded in a porous medium. As presented in Fig. 1, the direction of the x-axis is taken along the axis of the cylinder and the r-axis is considered to be in the radial direction. A uniform magnetic field of strength B_0 is applied in the radial direction, i.e., normal to the stretching cylinder. The stretching velocity $U_w(x)$ of the cylinder is assumed to be of the form $U_w(x) = U_0(x/L)$, where $U_0 > 0$ is a constant and L is the characteristic length. The surface of the stretching cylinder is subjected to a variable temperature $T_w(x) = T_\infty + T_0(x/L)^n$ and the ambient fluid temperature is T_∞. The induced magnetic field effects are considered to be negligible due to which magnetic Reynolds number is not taken. Moreover, it is supposed that the viscous dissipation and joule dissipation effects are present. Under all these assumptions, the boundary layer equations that govern the flow and heat transfer phenomenon are written as:

$$\frac{\partial}{\partial x}(ru) + \frac{\partial}{\partial r}(rv) = 0, \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial r} = \frac{\nu}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u}{\partial r}\right) - \frac{\nu}{K_p}u - \frac{\sigma B_0^2}{\rho}u, \tag{2}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial r} = \frac{k}{\rho c_p r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) + \frac{\nu}{c_p}\left(\frac{\partial u}{\partial r}\right)^2 + \frac{\sigma B_0^2}{\rho c_p}u^2 + \frac{\nu}{c_p K_P}u^2, \tag{3}$$

and the associated boundary conditions are:

$$\begin{aligned} &u = U_w(x),\ \ v = 0,\ \ T = T_w(x) \quad at\ \ r = a, \\ &u \to 0,\ \ T \to T_\infty \quad at\ \ r \to \infty, \end{aligned} \tag{4}$$

where (u,v) are the velocity components in the (x,r) directions, ν is the kinematic viscosity, ρ represents the fluid density, σ is the electrical conductivity of the fluid, B_0 is the uniform magnetic field, K_p is the permeability of the medium, k is the thermal diffusivity of the fluid, c_p is the specific heat at constant pressure, n is the temperature exponent and T is the temperature of the fluid.

Introducing the following similarity variables

$$\left.\begin{aligned} &\eta = \frac{r^2 - a^2}{2a}\left(\frac{U_w}{\nu x}\right)^{\frac{1}{2}}, \quad \psi = (U_w \nu x)^{\frac{1}{2}} a f(\eta), \\ &\theta = \frac{T - T_\infty}{T_w - T_\infty}. \end{aligned}\right\} \tag{5}$$

where η is the similarity variable, ψ denotes the stream function, f and θ are the dimensionless stream function and temperature respectively. The stream function is defined as

$$u = \frac{1}{r}\frac{\partial \psi}{\partial r} \quad \text{and} \quad v = -\frac{1}{r}\frac{\partial \psi}{\partial x}, \tag{6}$$

By substitution of (5) and (6), the Eq. (1) is automatically satisfied and Eqs. (2)–(3) are converted into following nonlinear ordinary differential equations

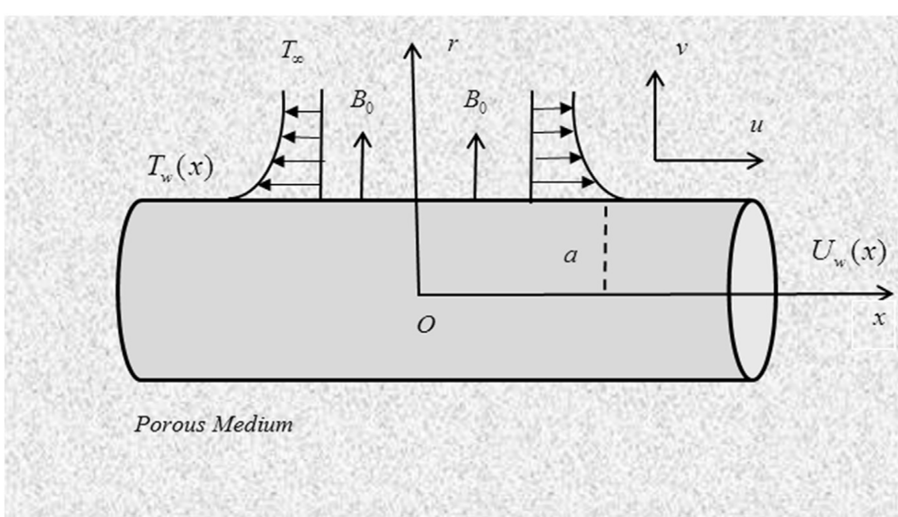

Fig. 1. Schematic representation of the considered problem.