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A finite line source model with Cauchy-type top boundary conditions for simulating near surface effects on borehole heat exchangers

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ABSTRACT

BHEs (borehole heat exchangers) are the most common shallow geothermal applications. By approximating the BHE as a line source, semi-analytical models can describe the heat exchange within the ground. These models though always assume prescribed temperature at the ground surface. This work presents a formulation which expands existing finite line source models by implementing a more general Cauchy-type top boundary condition and in this way, a better estimation of the heat fluxes at the ground surface. The new formulation is numerically verified and examined in a dimensionless analysis. It is demonstrated that the discrepancy to prescribed temperature settings is significant near to the ground surface, and it propagates deeper when groundwater flow is absent and when strong decoupling between the thermal regimes interacting at the land surface is assumed. The new approach shows to be suited especially for short BHEs, both for more flexible and accurate prediction of the ground thermal regime as well as for long-term analysis of technological performance.

1. Introduction

The utilization of low-enthalpy geothermal energy focuses on the shallow subsurface of some hundreds of metres depth. Most commonly, so-called ground heat exchangers or BHEs (borehole heat exchangers) are installed. These exchange heat with the ground by circulating a fluid through tubes installed in vertical boreholes. BHEs are usually connected to heat pumps defining the so-called GSHPs (ground source heat pump systems). During the last decade, the number of BHEs has significantly grown, especially in cities of central and northern Europe, the USA and China [1,2]. In 2015, worldwide annual utilization of GSHPs is estimated to reach 325 PJ [3].

With their number and density growing, there is also rising interest in improved simulation techniques to characterize and predict the thermal response in the ground. The most elementary simulation techniques are based on Kelvin's line source [4,5]. For example, the semi-analytical, infinite line-source solution is suitable for modelling seasonal energy exchange. It treats the ground as an initially isothermal, homogeneous medium, where heat is transported by conduction only. The BHE is approximated as a vertical line of infinite length embedded in the ground. Depending on the net thermal load on the system (net balance between heating and cooling loads), the line source creates a radial and expanding temperature gradient during system operation. After a typical operational life span (between 30 and 50 years), the extent can be up to tens of metres. On the long term, this may also affect neighbouring geothermal systems [6–10].

By approximating the ground as infinite conductive system, any further processes and boundary conditions are neglected. These, however, have often shown to be crucial, and thus advanced line source solutions have been proposed. Recent advancements in analytical BHE modelling focus on improved expressions for more efficient computer-based implementation, short term simulation and high time-resolution of operation with discontinuous heat extraction or injection [11–14]. Li and Lai [15,16] develop techniques to deal with anisotropy and heterogeneity of the surrounding medium with analytical models. One process is advective heat flux stimulated by horizontal groundwater flow. In fact, many BHEs are installed in dynamic aquifers, so that even in a conduction-dominated environment, advection may have a remarkable influence on the evolution of ground temperatures around BHEs [17]. Thermal anomalies are deformed to heat or cold plumes elongated in downstream groundwater flow direction

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[18,19]. Sutton et al. [20] and Diao et al. [21] showed how to include horizontal advection in the so-called moving line source [4,22]. Molina-Giraldo et al. [23] included mechanical dispersion, and Molina-Giraldo et al. [24] introduced a MFLS (moving finite line source). Even if BHEs are installed in long boreholes, the simulation as finite lines is more accurate and especially axial effects at the borehole toe can be considered [25–28]. Thermal anomalies would be overestimated when modelling a BHE as infinite, whereas the calculation error increases the shorter the BHE is.

Existing BHE models implement the top boundary of the line source as Dirichlet-type (1st-kind) boundary condition, either as fixed [6,21,24] or time-dependent temperature [29,30]. Even if formulations with 1st-kind condition can consider variations in space and time, they represent general simplifications of the situation in the field. For example, when the ground surface temperature is assumed to be equal to the atmospheric temperature, the isolating effect of snow cover is ignored [31,32]. Generally, the SAT (surface air temperature) is more accessible and better monitored than the GST (ground surface temperature). Existing models often take the SAT as an approximation for the GST neglecting the different behaviours and coupling of both temperature fields [33]. As an alternative, the Cauchy-type or 3rd-kind boundary condition assumes a linear heat transfer rate between the subsurface's upper layer and the atmosphere. This boundary formulation represents, in a more realistic manner, the thermal regime near to the ground and hence constitutes a more general approach for this boundary-value problem [4,34].

Formulations with Cauchy-type boundary conditions are common in land-surface models (e.g. Refs. [35,36]). Analytical models have been developed to study land-atmosphere processes but without including the coupled effect of shallow geothermal systems (e.g. Refs. [37,38]). Cauchy-type boundary conditions are also applied for simulating shallow geothermal applications such as ATES (aquifer thermal storage systems), energy piles and very-shallow helical ground heat exchangers [39–43]. However, for these applications, even simple problems are exclusively solved by numerical techniques.

This work proposes an alternative analytical model framework that can handle Cauchy-type boundaries for analysing the thermal effects of BHEs in the ground. It is fast, can incorporate horizontal groundwater flow and simulate transient thermal conditions. The model is numerically verified and sensitivity analyses are provided to reveal the role of conduction, advection and boundary settings.

2. Methodology

Fig. 1 schematically shows a vertical profile with a BHE implemented in a 3-D (three-dimensional) semi-infinite space. The BHE is approximated as a line source with constant heat extraction/injection rate q and length H. It is also assumed constant, isotropic and homogeneous flow and transport properties of the porous medium (e.g. Refs. [21,44,45]). This homogenization strategy has shown to be a good approximation specially in conduction-dominated systems (e.g. Refs. [28,46]). For the system depicted in Fig. 1, the transient temperature distribution is given by the heat transport equation (e.g. Refs. [4,47]):

$$\frac{\partial T}{\partial t} + \boldsymbol{v} \cdot \nabla T = \nabla \cdot (a \nabla T) \tag{1}$$

where $\boldsymbol{v}$ is the effective thermal velocity vector with magnitude $v = D\frac{c_w}{c}$. The Darcy flux, D, is assumed uniform and steady. This may be a strong assumption depending on the specific case. However, it is not expected that v plays an important role in the temperature response close to the ground surface. In other applications

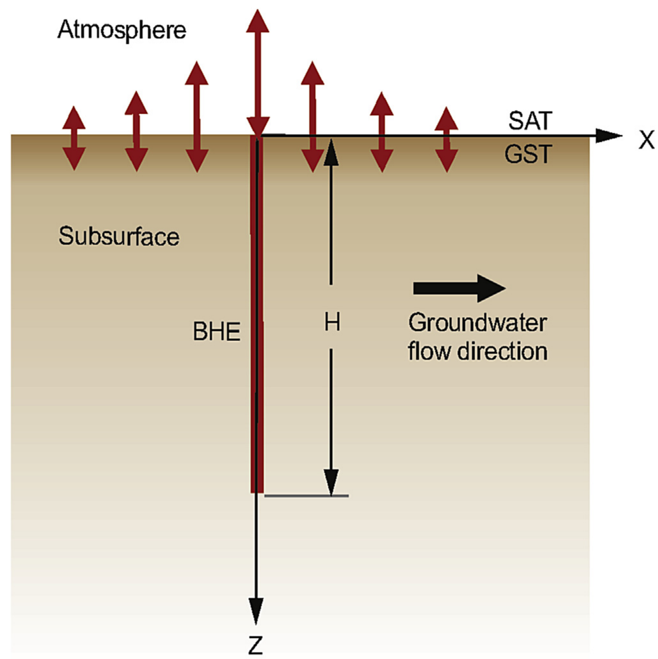

Fig. 1. BHE represented as a line source in a semi-infinite space with decoupled surface air temperature (SAT) and ground surface temperature (GST). The red arrows sketch heat fluxes at the ground surface. Downward fluxes occur when SAT > GST.

characterized by complex hydrogeological conditions, numerical models are probably the only way around (e.g. Refs. [48,49]). In Eq. (1), the parameter a represents the effective thermal diffusivity given by $a = \frac{\lambda}{c}$ where λ and c are the effective conductivity and effective volumetric heat capacity of the porous medium, respectively.

A Neumann-type BC (boundary condition) represents the BHE by ([4,5])

$$\lim_{r_b \to 0} 2\pi r_b \lambda \frac{\partial T}{\partial r_b} = -q, \quad 0 \le z \le H \tag{2}$$

where r_b is the borehole radius. A Cauchy-type BC is imposed at the ground surface to account for a linear heat transfer between the subsurface and the atmosphere. In this BC, the heat flux is proportional to the difference between the GST (ground surface temperature) T and the temperature of the medium above the ground T_s (e.g. SAT). This temperature as well as the initial temperature field are equal to zero in this homogeneous boundary value problem (Fig. 1):

$$\lambda \left.\frac{\partial T}{\partial z}\right|_{z=0} = -I[T - (T_s = 0)] \;\text{ or }\; \left.\frac{\partial T}{\partial z}\right|_{z=0} = -hT \text{ with } h = \frac{I}{\lambda} \tag{3}$$

In Eq. (3), I [$W\ m^{-2}\ K^{-1}$] is the surface heat transfer coefficient [4]. The coefficient h [m^{-1}] can be regarded as the coupling coefficient between GST and T_s [33]. Theoretically, h can range between 0 (no coupling) and infinite (perfect coupling), representing either full insulation or a Dirichlet-type BC at the ground surface, respectively. The assumptions of an initially zero temperature field as well as $T_s = 0$ may be relaxed by superposing the associated nonhomogeneous boundary value problem as indicated in Refs. [29,30].

Compared with the traditionally prescribed Dirichlet-type BC ($T = T_s$), Eq. (3) offers a more flexible and realistic representation of near-surface conditions influencing the thermal regime around