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Analytical closed-form model for predicting the power and efficiency of Stirling engines based on a comprehensive numerical model and the genetic programming

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ABSTRACT

High accuracy and simplicity in use are two important required features of thermal models of Stirling engines. A new numerical second-order thermal model was presented through the improvement of our previous modified-PSVL model in order to have an elevated accuracy. The modified-PSVL model was modified by considering a non-isothermal model for heater and cooler. Then, the model called as CPMS-Comprehensive Polytropic Model of Stirling engine, was used to simulate the GPU-3 Stirling engine, and the obtained results were compared with those of the previous thermal models as well as the experimental data. For the sack of the simplicity, the combination of the CPMS model and genetic programming was employed to generate analytical closed-form correlation. In this regards, a comprehensive data bank of results of the CPMS was constructed and exported to the GP tool and analytical expressions of the power, efficiency, and polytropic indexes were obtained. It was shown that the analytical correlations not only had the same accuracy as the CPMS model, but also, it can be simply used without difficulties of numerical models. The CPMS and its out coming analytical expressions, predicted the power and efficiency of the GPU-3 Stirling with +1.13% and +0.45 (as difference), respectively.

1. Introduction

Designing Stirling engines requires to have thermal models that are able to predict thermal performance of Stirling engines with reasonable accuracy based on thermal and geometrical specifications of these engines. Hence, thermal models for predicting power and thermal efficiency of Stirling engines are highly desired by engineers. There are several categories of thermal models of Stirling engines, including empirical [1–5] and analytical models [6–28]. Analytical models itself are classified into two sub-categories, including the closed-form numerical models. The closed–form models [6–14] are also called as the first-order models, while numerical models include second order zero dimensional [15–25] and third-order CFD [25–28] models. Among the thermal methods, the most accurate models are the numerical ones. Descriptions of these categories of thermal models were given in more detail by authors in Ref. [23]. In summary, first-order closed-form models are mostly simple thermodynamic models that easily estimate thermal performance of Stirling engine based on the operating parameters; however, they usually suffer from lack of enough accuracies required by designers. Although, the CAFS model [25] was developed to simulate thermal performance of Stirling engine with sufficient accuracy, but it is still unable to consider the effects of detail structures of the engine such as geometric specifications of heat exchangers. One the other side, there is 3D numerical CFD models with more accuracy, but they are very complicated and time consuming. On the other hand, they are developed for a special type of engine, and their results cannot be extended to all types of Stirling engines. In addition, due to complexity of phenomena and structure, it is very difficult to make a comprehensive CFD model, hence a simplified CFD model of the real engine is usually made; therefore, the accuracy of the model is reduced.

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Second order models are numerical zero dimensional models that are developed based on the time dependence of governing equations while the spatial dependence of governing equations is ignored. In these models, a system of boundary value ordinary differential equation, ODE, with respect to crank angle (time) is developed based on mass and energy equations of engine compartments. Then, the system of equations is converted into an initial value problem (see Ref. [15]), and they are solved numerically usually with fourth order Runge–Kutta method. As a pioneer generation of second order models, Urieli and Berchowitz [15] developed a numerical thermal model based on adiabatic expansion/compression processes. Furthermore, they modified their ideal adiabatic model to a new model, called the Simple analysis, in order to implement the effect of non-ideal heat recovery of the regenerator, non-ideal heat transfer in cooler and heater, and pressure drops in heat exchangers [15]. Later, original simple analysis of Urieli and Berchowitz [15] has been modified by a number of researchers [16–26] to include various loss mechanisms of real engines that were missed in the original Simple model [15]. In a more recent work, Babaelahi and Sayyaadi [25] developed a new thermal model called as PSVL (polytropic analysis of Stirling engine with various losses). In their work, adiabatic or isothermal assumption of expansion/compression processes of previous works [15–24] was substituted with polytropic processes. In addition, they integrated the effects of the shuttle conduction heat loss and mass leakage into differential equations and corrected results of the numerical solution for the effects of non-ideal heat transfer and pressure drops in heat exchangers, finite motion of the piston (based on finite speed thermodynamic model), mechanical friction and gas throttling. Later, Babaelahi and Sayyaadi [26] modified their original PSVL model called as the modified PSVL based on coupling the convective heat transfer mechanism around the cylinder wall to the polytropic model for more accurate estimation of polytropic indexes. Furthermore, the linear temperature profile of gas in the regenerator was substituted with an exponential temperature profile. They demonstrated that the modified PSVL model has superior accuracy to all previous second and third-order models [26].

The objective of this paper is presenting a closed-form first-order model which has the same accuracy of latest numerical models as well as enables to consider details constructing and operating parameters of Stirling engines. In this regard, in the first step, the previous numerical model of authors called as modified PSVL was corrected to be able to consider non-isothermal heater and cooler and more accurate model called as CPMS-Comprehensive Polytropic Model of Stirling engine was presented. In all previous numerical second order models [15–26], gas temperature in heater and cooler was assumed to be isothermal, as it was difficult to consider a non-isothermal profile of real cases in the numerical solution. It was due to the increasing number of unknown parameters that were more than the number of equations in fourth order Runge–Kutta solution of ODE. In the present model, based on a procedure comes from thermal modeling of heat exchangers, the temperature profiles of the gas in the heater, and the cooler were estimated and used in the numerical solution. This profile affected the boundary conditions of ODEs. On the other hand, since the thermohydraulic model of heat exchangers is based on thermophysical gas properties at the average temperature, temperature distribution profile affects the average temperature and consequently, the thermal model. Therefore, the results of the numerical solution were altered from those models that use the isothermal temperature profile for these heat exchangers. Then, the outputs of the differential model were corrected for effects of power losses due to finite motion of the pistons, throttling of gas at the inlet/outlet of the engine compartments, the mechanical friction, and longitudinal conduction through the regenerator wall were considered in the model in a similar manner to [24–26]. The CPMS model was applied to a benchmark Stirling engine called as the GPU-3 engine and its results were compared to previous models as well as the experimental results.

In the next step, the non-dimensional analysis of Stirling engine was performed and non-dimensional numbers of Stirling engine were presented and engine power, thermal efficiency, and polytropic indexes were evaluated at various non-dimensional numbers using the numerical model. Therefore, an extensive database was created using the numerical model. This database contains magnitudes of the power, thermal efficiency, and polytropic indexes at different values of non-dimensional parameters. The objective of the paper was correlated these outputs as functions of the input parameters (no-dimensional numbers). These correlations give the first order like thermal model, but with the same accuracy of the most accurate numerical model. In this regards, the generated database was exported to the genetic programming (GP) tool, and suitable analytical correlations were constructed for Stirling engine. These equations correlate the power, thermal efficiency, and polytropic indexes of Stirling engine as functions of non-dimensional parameters. Comparison between new analytical correlations and experimental results as well as the new numerical model was performed. The accuracy of the analytical model as well as the new numerical models was emphasized. Moreover, effects of variation in non-dimensional parameters on thermal performance of the GPU-3 engine was studied. The analytical model that was constructed for the GPU-3 engine having the Rhombic drive was also extended to engines with the Ross-Yoke driving mechanism as well. Therefore, the obtained analytical model can assist designers of Stirling engines to evaluate thermal features of these engines as functions of operation and constructing parameters of the engine with reasonable accuracy without the need to deal with complex numerical models.

2. Modeling

In the CPMS thermal model, in a similar manner to other numerical second order models [15–26], the engine was divided into five compartments, including heater, Kooler (cooler), regenerator, expansion, and compression compartments, as shown in Fig. 1. Conservation equations of mass and energy were implemented for each compartment considering the fact that mass and energy transfer phenomena in these compartments are unsteady, in which variables are changed with crank angle. In developing the system of differential equations, derivation of parameters in terms of crank angle (θ), was denoted by $d_{:}$ i.e. $d \equiv \frac{d}{d\theta}$ hereinafter.

By the implication of the unsteady governing equations of mass and energy for five engine's compartments and considering the shuttle effect and mass leakage term (see Ref. [26]) on the one hand, and implication of the proper boundary conditions between the compartments, on the other hand, an ODE system similar to the system of equation in the original PSVL model [25] was obtained. This ODE system included shuttle-effect heat loss and mass leakage can be found in Ref. [26].

In addition, in the modified PSVL model [26], an exponential temperature profile was suggested for temperature in the regenerator; therefore, the average temperature of gas in the regenerator was calculated as follows [26]: